DESIGN CRITERIA FOR LIGHTLY LOADED INFLATED STRUCTURES IN A NEAR-PLANETARY ATMOSPHERE

By Peter G. Niederer

ARC-R-246

Prepared under Contract No. NAS7-427 by
ASTRO RESEARCH CORPORATION
Santa Barbara, California

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES (contd)

LIST OF TABLES

DESIGN CRITERIA FOR LIGHTLY LOADED INFLATED STRUCTURES IN A NEAR-PLANETARY ATMOSPHERE

By Peter G. Niederer
Astro Research Corporation

SUMMARY

A requirement exists to erect structures in space which are too bulky to be transported to their operating environment in their full size. The problem can be solved by inflating a lightly loaded membrane, and the simplest solution is offered by carrying a pressurization medium directly within the structure. The difference of state of this medium between the packed condition at the launching site and the deployed condition in the operating environment can be used to deploy the body and then to rigidize it to perform a given task. Two near-earth environments and one near Mars are chosen for closer investigation. An exact knowledge is required of the equilibrium temperature a body attains due to various radiative sources in space. Temperature can be regulated to a desired level by the appropriate choice of the solar absorptivity and emissivity of the body surface. Various pressurization substances are examined and an investigation is made regarding their abilities to develop a certain pressure by varying the quantity used to inflate a given volume. An analysis is made to minimize the weight of an inflated body for a given load carrying task. The whole theory is exemplified by its application to the bracing system of an X-brace Stokes flow decelerator as described in Report ARC-R-236 (ref. 8).

INTRODUCTION

Structures used in space are often too bulky to be transported in their full size to their operating environment. They have to be folded or disassembled in order to be packaged into a transporting vehicle. Correspondingly, they have to be unfolded or assembled after their arrival at the operating point in space. The general requirements governing the design of such structures can be summarized into the following three design goals:

1. Minimum weight
2. Minimum packing volume
3. Simple deployment scheme, if possible self-actuating

An attractive solution to the problem is the inflated structure formed by a thin membrane if its form is simple and if its operational loads are small. The deployment scheme is simplest if the medium pressurizing the structure is carried within its enclosure. The difference of state of this pressurization medium between the packaged condition at the launching site and its deployed condition in the operating environment can be used for both purposes, i.e., to deploy the structure and then to stabilize its shape as required to perform a given task.

There are numerous applications of this system wherever the requirements of minimum weight and minimum packing volume are at a premium: beam antennas, support for large space stations, transportation vehicles on other planets (blimps, inflated aircraft), deployment aids for parachutes operating at high altitudes or for reentry deceleration (beams, toroids, spheres) and many others.

The present report limits its investigation to a very simple structure, a <u>straight cylindrical column</u> of various aspect ratios. The column is assumed to carry a <u>compressive load</u> P at its ends which are apart by the column's length L. Three points in the higher atmosphere of the planet earth (altitudes 80 km and 100 km) and Mars (altitude 110 km) are chosen as the column's operating environment.

The design problem of an inflated structure can be divided into three main topics.

1. Proper inflation requires the knowledge of the temperature at which inflation can take place. Exposure to the sun and the radiation due to the nearby planet (albedo, infra-red radiation) tend to heat the body up, which itself radiates heat into its surrounding space. A steady state of heat in-and-output to the body is reached at an equilibrium temperature to be determined below. If the body is moving, there is a further heat exchange expected in the form of forced convection from the body to the surrounding atmosphere. The following investigation shows that the influence of this forced convection term is small, it will, however, be included for further calculations. It will be seen that the level of the desired equilibrium temperature can be largely regulated by the proper choice of the ratio of solar absorptivity to emissivity of the surface of the beam.

2. The choice of the pressurization medium depends on its ability to develop a certain pressure at the expected equilibrium temperature. The variation of the amount of pressurization medium to inflate a given volume is characterized by its "packing volume ratio", which is the ratio of the volume taken by the folded structure (to be packaged into a payload compartment) to the fully deployed volume of the structure. Various suitable pressurizing media are examined. Their range of application is established for a proper self-inflation of the beam.

3. It is required that the given operational task of the beam be performed with a minimum amount of weight of the whole assembly. The minimum weight is largely limited by the availability of a skin material of minimum practical thickness, such as a Mylar film. Stress limitations are apparently only secondary limits to the application considered here.

This report closes with Appendix E describing an application of the theory to a high altitude parachute (ref. 8). Its canopy is deployed and rigidized by an X-shaped bracing system formed by a cross-wise arrangement of two inflated slender beams. The pressurization of these beams is discussed.

SYMBOLS

The Heat Balance of an Inflated Beam

A	$[m^2]$	total surface area of body
A_B	$[m^2]$	composite area exposed to planetary radiation (albedo, infrared)
$AR = \frac{\ell}{r}$		aspect ratio
A_S	$[m^2]$	composite area exposed to sun radiation
B		average albedo factor expressing the ratio of reflected solar energy to impinging solar energy
C_P	$\left[\frac{kcal}{kg°K}\right]$	specific heat
c	$\left[\frac{kcal}{h°K^4}\right]$	heat conduction coefficient
$d = 2r$	$[m]$	column diameter
E		average infrared radiative factor expressing the emitted infrared radiation of the planet as a fraction of the solar constant
h	$\left[\frac{kcal}{h°Km^2}\right]$	film coefficient of heat transfer
ℓ	$[m]$	column length
r	$\left[\frac{kcal}{h°K^4}\right]$	radiative transfer coefficient
S	$\left[\frac{kcal}{m^2 hr}\right]$	solar constant (average)

T	$[^\circ K]$	absolute temperature
t	$[h]$	time
V	$[m^3]$	beam volume (when inflated)
α		solar absorptivity
Δ	$[m]$	thickness of skin
ε		emissivity
ρ	$\left[\frac{kg}{m^3}\right]$	density
$\sigma = 4.96 \cdot 10^{-8}$	$\left[\frac{kcal}{m^2 h ^\circ K^4}\right]$	Boltzmann constant

Subscripts:

a	atmospheric temperature
g	pressurization gas
s	skin

The pressurization of an Inflated Structure

m	$[kg]$	total mass of pressurization medium
p	$\left[\frac{N}{m^2}\right]$	pressure $p\left[\frac{N}{M^2}\right] \cdot 7.5 \cdot 10^{-3} = p[\text{mm Hg}]$ $p\left[\frac{N}{m^2}\right] \cdot 1.45 \cdot 10^{-4} = p[\text{psi}]$
R	$\left[\frac{Nm}{^\circ K kg}\right]$	gas constant
T	$[^\circ K]$	absolute temperature
V	$[m^3]$	volume occupied by pressurizing medium
$v = \frac{V_p}{V_d}$		packing volume ratio

Subscripts:

d	deployed condition
p	packed condition (at earth's surface)
$\bar{p}$ $\bar{\rho}$ $\bar{R}$	conditions of state of mixture
1, 2	components of pressurizing medium mixture

The Minimum Weight of the Beam

E	$\left[\frac{N}{m^2}\right]$	modulus of elasticity
$g = 9.81$	$\left[\frac{m}{s^2}\right]$	gravity constant (earth)
$K = \frac{1}{k}$		safety factor against local buckling
ℓ	$[m]$	column length
P	$[N]$	compressive load
p	$\left[\frac{N}{m^2}\right]$	internal column pressure
r	$[m]$	column radius
w	$[N]$	total weight of column
Δ	$[m]$	membrane thickness
ρ	$\left[\frac{kg}{m^3}\right]$	density
σ	$\left[\frac{N}{m^2}\right]$	hoop stress

Subscripts:

g		pressurization gas
s		skin

Appendix B

A_1	$[m^2]$	heated area
A_2	$[m^2]$	radiator area
F		shape factor of radiative heat transmitter of element dA_1 with respect to radiator
H	$[km]$; $[m]$	altitude above planet's surface
h	$[km]$; $[m]$	altitude above flat disc radiator
Q	$\left[\frac{kcal}{h}\right]$	heat flux from A_2 to A_1
R	$[km]$; $[m]$	radius of disc radiator
R_p	$[km]$; $[m]$	radius of planet
s	$[m]$	distance between A_1 and A_2
T	$[°K]$	absolute temperature
β	$[°]$	angle with respect to normal direction of area
$\sigma = 4.96 \cdot 10^{-8}$	$\left[\frac{kcal}{m^2 h °K^4}\right]$	Boltzmann constant

Appendix D

h	$\left[\frac{kcal}{h°Km^2}\right]$	film coefficient of heat transfer
k	$\left[\frac{kcal}{h°Km}\right]$	thermal conductivity
$Nu_d = \frac{hd}{k}$		Nusselt number
$P_r = \frac{C_p \eta}{k}$		Prandtl number
R	$\left[\frac{J}{°Kkg}\right]$	gas constant
$Re_d = \frac{Vd\rho}{\eta}$		Reynolds number
v	$\left[\frac{m}{s}\right]$	velocity
η	$\left[\frac{kg}{msec}\right]$	dynamic viscosity

THE HEAT BALANCE OF AN INFLATED COLUMN

The Heat Balance Equation

A cylindrical column of length ℓ and radius r is given. It is inflated by means of a gaseous medium at a pressure p_g and temperature T_g. The column itself is formed by a thin film which has a temperature T_s. Its outer surface is characterized by an emissivity ϵ and a solar absorptivity α.

The body is immersed in a rarefied atmosphere of temperature T_a at an altitude H above the planet's surface. It is assumed that the beam is moving through this atmosphere at a velocity v.

Two heat balance equations can be formulated for the skin and the gaseous interior of the beam. Each time the sum of the received heat contributions and of the emitted heat contributions are equated to the considered part's change in heat capacity.

$$\alpha S A_s G_2 + \alpha B S A_B G_1 G_2 + \epsilon E S A_B - \epsilon\sigma A T_s^4 - r\left(T_s^4 - T_g^4\right) - c\left(T_s - T_g\right) - hA\left(T_s - T_a\right) = \Delta A \rho_s C_{ps} \cdot \frac{dT_s}{dt} \quad (1)$$

$$r\left(T_s^4 - T_g^4\right) + c\left(T_s - T_g\right) = V \rho_g C_{pg} \cdot \frac{dT_g}{dt} \quad (2)$$

For the detailed meaning of the symbols consult the list of symbols page 4.

Equation (1) for the skin shows the following terms in the same sequence:

- parallel sun radiation
- diffuse albedo radiation from the planet (reflected sun radiation)
- diffuse infrared radiation from the planet
- skin radiation outwards
- radiative heat exchange between the skin and the pressurizing gas
- convective heat exchange between the skin and the pressurizing gas
- forced convective heat exchange with the atmosphere
- skin heat capacity

The terms in equation (2) for the gas represent:

- radiative heat exchange from skin
- convective heat exchange from skin
- gas heat capacity

If steady state conditions are assumed, then

$$\frac{dT_s}{dt} = \frac{dT_g}{dt} = 0$$

and according to equation (2):

$$T_s = T_g$$

The whole body, skin and pressurizing gas, is isothermal.

Appendix A demonstrates with a simple example, that this steady state temperature is reached within a short time, e.g., below one minute.

Equation (1) can now be written as

$$T_s^4 + T_s\left[\frac{h}{\epsilon}\cdot\frac{1}{\sigma}\right] = \frac{s}{\sigma}\left[G_2\cdot\frac{\alpha}{\epsilon}\left(\frac{A_s}{A} + B\cdot\frac{A_B}{A}\cdot G_1\right) + E\cdot\frac{A_B}{A}\right] + T_a\left[\frac{h}{\epsilon}\cdot\frac{1}{\sigma}\right] \qquad (3)$$

This equation has the form

$$T_s^4 + T_s k = T_{so}^4 + T_a\cdot k = K$$

k and K being constants for a given situation and

$$T_{so}^4 = \frac{s}{\sigma}\left[G_2\cdot\frac{\alpha}{\epsilon}\left(\frac{A_s}{A} + B\cdot\frac{A_B}{A}\cdot G_1\right) + E\cdot\frac{A_B}{A}\right] \qquad (3A)$$

being the equation for a temperature equilibrium without taking into account the convective heat transfer term.

Input Data

Three "time of day" cases are investigated, characterized by the following simplified body-planet-sun positions:

"morning"	column at right angles to the sun-planet line
"midday"	column on the sun-planet line, and between the sun and the planet
"midnight"	column on the sun-planet line, but behind the planet

The column's orientation with respect to the planet is given by three investigated positions which are at right angles to each other for the three considered daytime cases. Figure 1 shows:

"position 1"	column axis parallel planet-body line
"position 2"	column axis at right angle to planet-body line, and parallel planet-sun line.
"position 3"	column axis at right angle to planet-body line, and at right angle to planet-sun line.

For the solar constant S an average value is taken. The intensity of the albedo and infrared radiation from the planet is expressed in average fractions B and E of the solar radiation, according to reference 1, 22 - 105.

The factors G_1 and G_2 characterize the time-of-day. A_S and A_B represent the effective surface areas exposed to sun radiation and planetary radiation respectively. They are usually sums of surface area sections multiplied by their shape factors describing the heat exchange between the heat source and the particular surface area section. Appendix B gives a more detailed derivation of A_B and A_S for the beam of various aspect ratios and at various positions.

A summary of the parameters chosen for this analysis is given in table I. The main parameter of the problem is the ratio of the solar absorptivity to the emissivity $\frac{\alpha}{\epsilon}$. For the program the following simplified scheme is adopted covering a wide range of possible α/ϵ = values:

α	0.1	1.0	1.0
ϵ	1.0	1.0	0.1
$\frac{\alpha}{\epsilon}$	0.1	1.0	10

Whereas a small α/ϵ corresponds to a white-lead-like surface, large α/ϵ can be achieved using metal surfaces or surfaces with optically thin coatings. An α/ϵ = 1.0 represents the case of the ideal black body. Appendix C tabulates actual data on α and ϵ , as given by the references 6 and 7.

The temperature equilibrium equation (3A), due solely to radiative heat transfer to and from the body, depends on surface area ratios only. The inclusion, however, of a term describing a heat transfer contribution due to forced convection (in eq. (3)) requires an indication of the absolute size of the body due to the dependence of the heat film coefficient h on Reynolds number and Nusselt number. Depending on the flow conditions considered a column diameter $d = 1m$ or a column length $\ell = 1m$ is assumed and aspect ratio effects are neglected. Appendix D presents a short analysis which leads to the numerical values of h used in this report and tabulated in table I.

Summary of the Results

The complete equilibrium temperature analysis has been programmed to be processed electronically on a General Electric GE-235 digital computer using the time sharing system. The most prominent results are graphically compiled on the following figures:

Figures 2 and 3: Variation of aspect ratio for two positions of the column

Figures 4, 5 and 6: Variation of time-of-day, correction due to the forced convection term for the three cases for the earth and Mars.

The following general conclusions can be drawn:

1. The most important parameter throughout the problem is α/ϵ. As long as the sun can be used as a heat source, a wide range of possible equilibrium temperature is offered by varying α/ϵ of the beam surface.

2. The influence of AR is only significant for the case when the end surfaces of the beam are comparable in size to the cylindrical surface. This suggests that the influence can be neglected for $AR > 20$.

3. The influence of the forced convection term is usually small for "midday" cases and somewhat more pronounced for "morning" cases.

4. A comparison of temperatures at the two investigated earth altitudes shows a surprisingly low temperature increase due to forced convection at the lower altitude.

PLANETARY PARAMETERS:

Planet	Solar const. S $\left[\frac{k\ cal}{m^2\ hr}\right]$	B	E
Earth	1200	0.39	0.15
Mars	519	0.148	0.37 *)

*) based on a surface temperature 250°C

ATMOSPHERIC PARAMETERS:

Planet	H [km]	V $[\frac{m}{s}]$	ρ $[\frac{kg}{m^3}]$	T_a [°K]	Parallel flow (pos.1)			Cross flow (pos.2, 3)		
					Re_ℓ	Nu_ℓ	h $[\frac{k\ cal}{h\ °k\ m^2}]$	Re_d	Nu_d	h $[\frac{k\ cal}{h\ °k\ m^2}]$
Earth	80	80	$1.99 \cdot 10^{-5}$	180	13.5	1.07	0.1486	13.5	2.20	0.306
Earth	100	250	$4.974 \cdot 10^{-7}$	210	0.85	0.272	0.0438	0.85	0.87	0.140
Mars	110	80	$2.19 \cdot 10^{-5}$	180	7.2	0.79	0.0972	7.2	1.72	0.233

"TIME-OF-DAY" PARAMETERS:

Daytime	G_1	G_2
Morning	0.5	1.0
Midday	1.0	1.0
Midnight	0	0

Surface parameters: See Appendix B

Table 1: Choice of Parameters

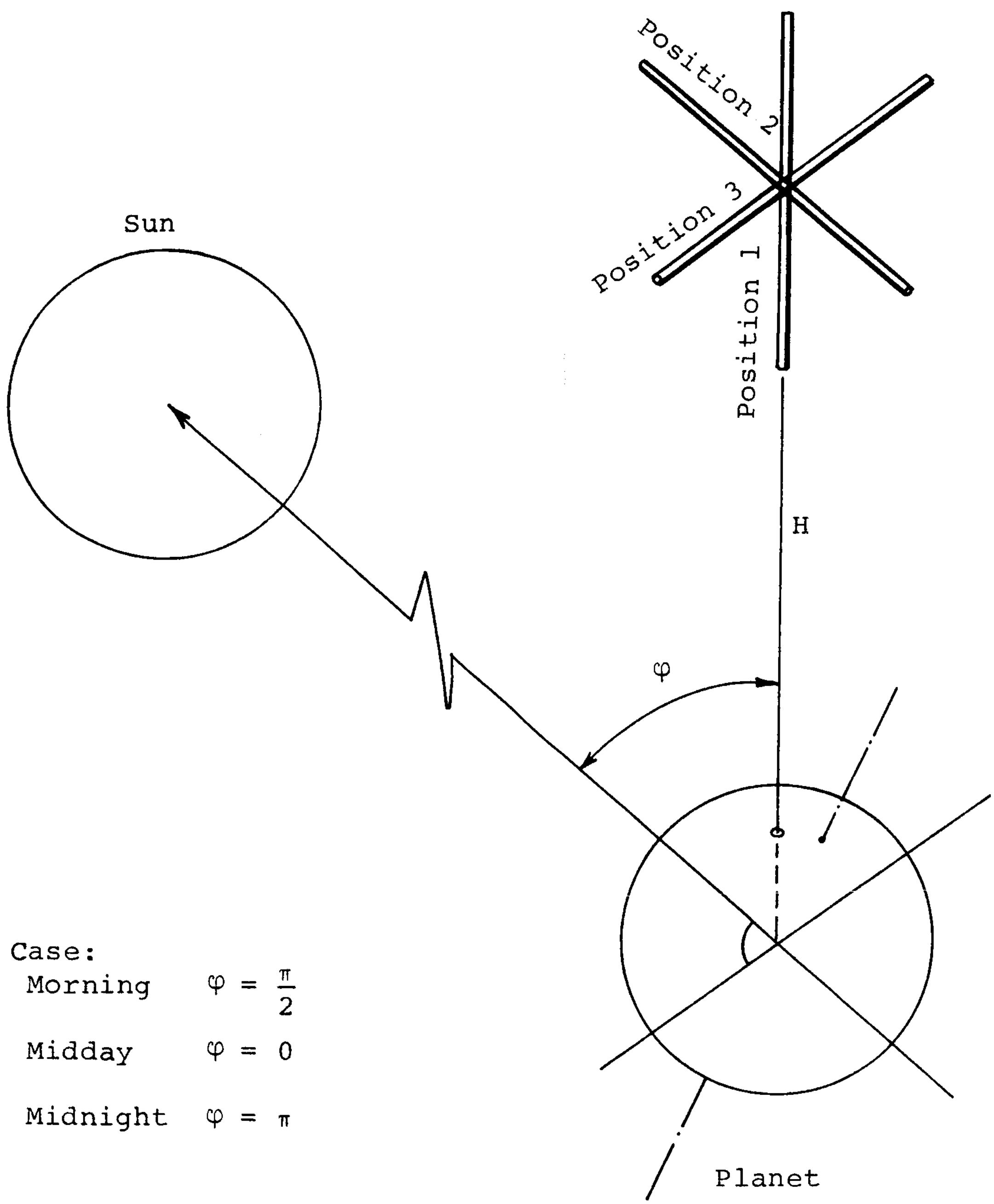


Figure 1. Definition of the Beam Positions

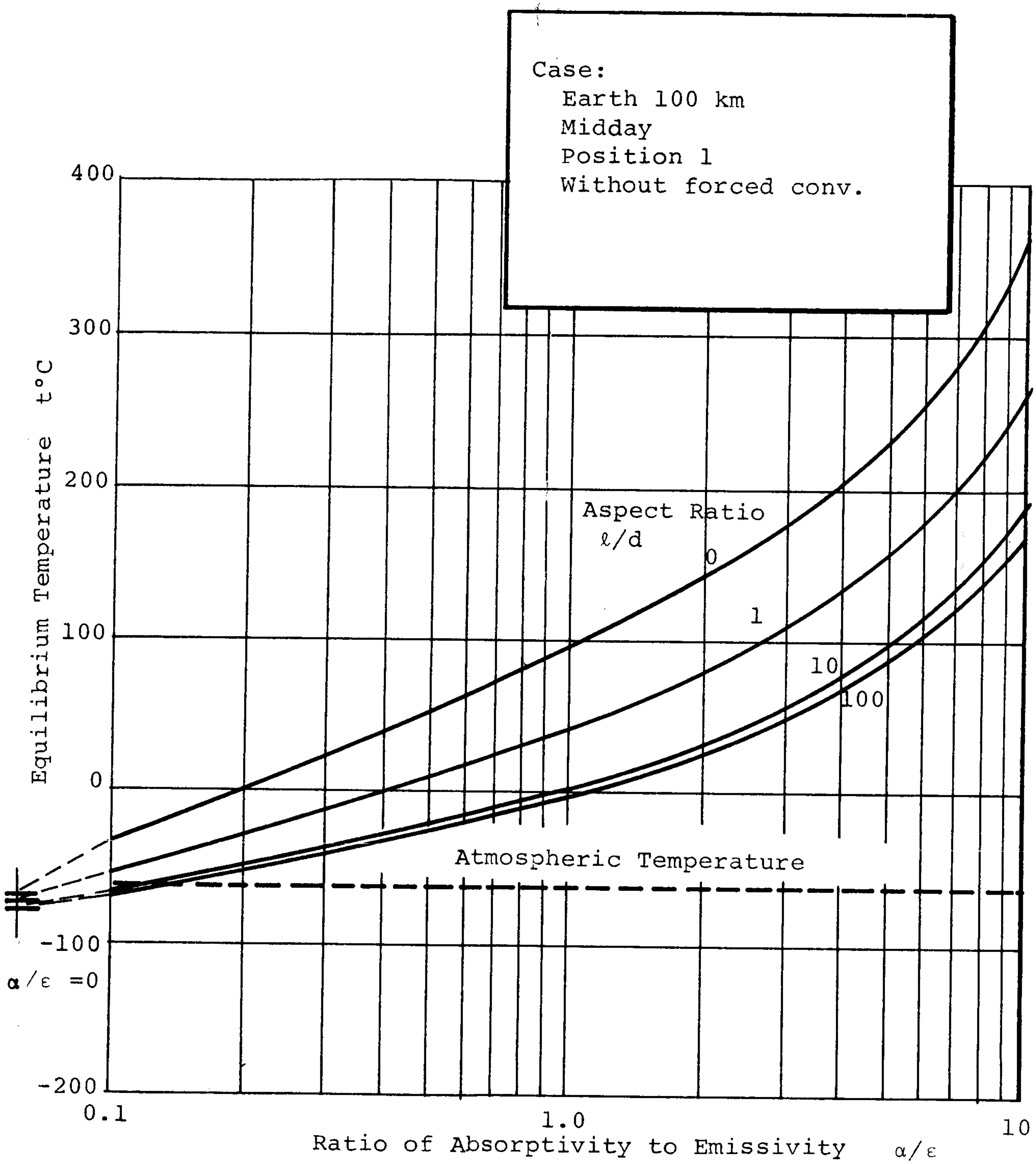


Figure 2. Equilibrium Temperatures for an Inflated Column: Position 1: Variation of Aspect Ratio

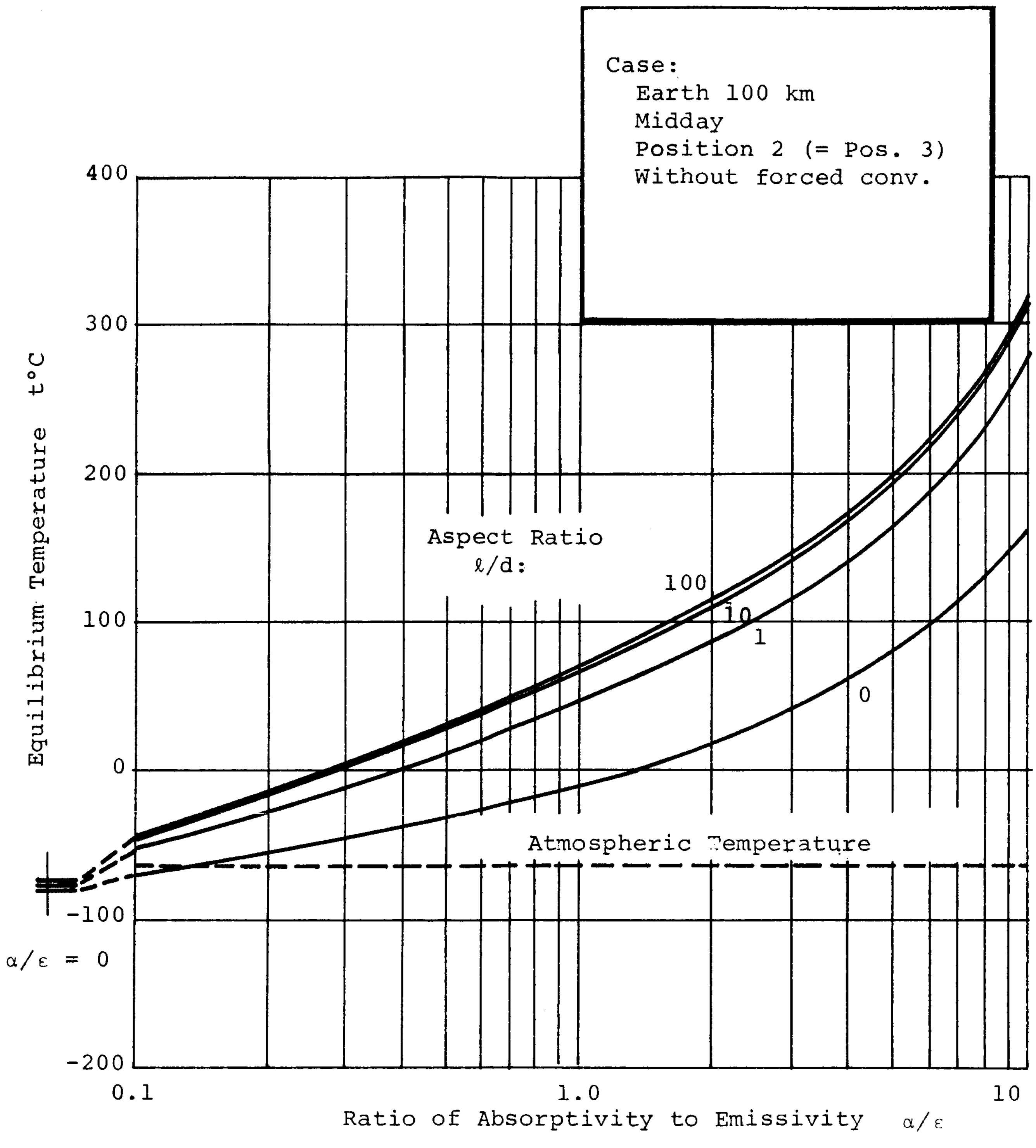


Figure 3. Equilibrium Temperatures for an Inflated Column: Position 2: Variation of Aspect Ratio

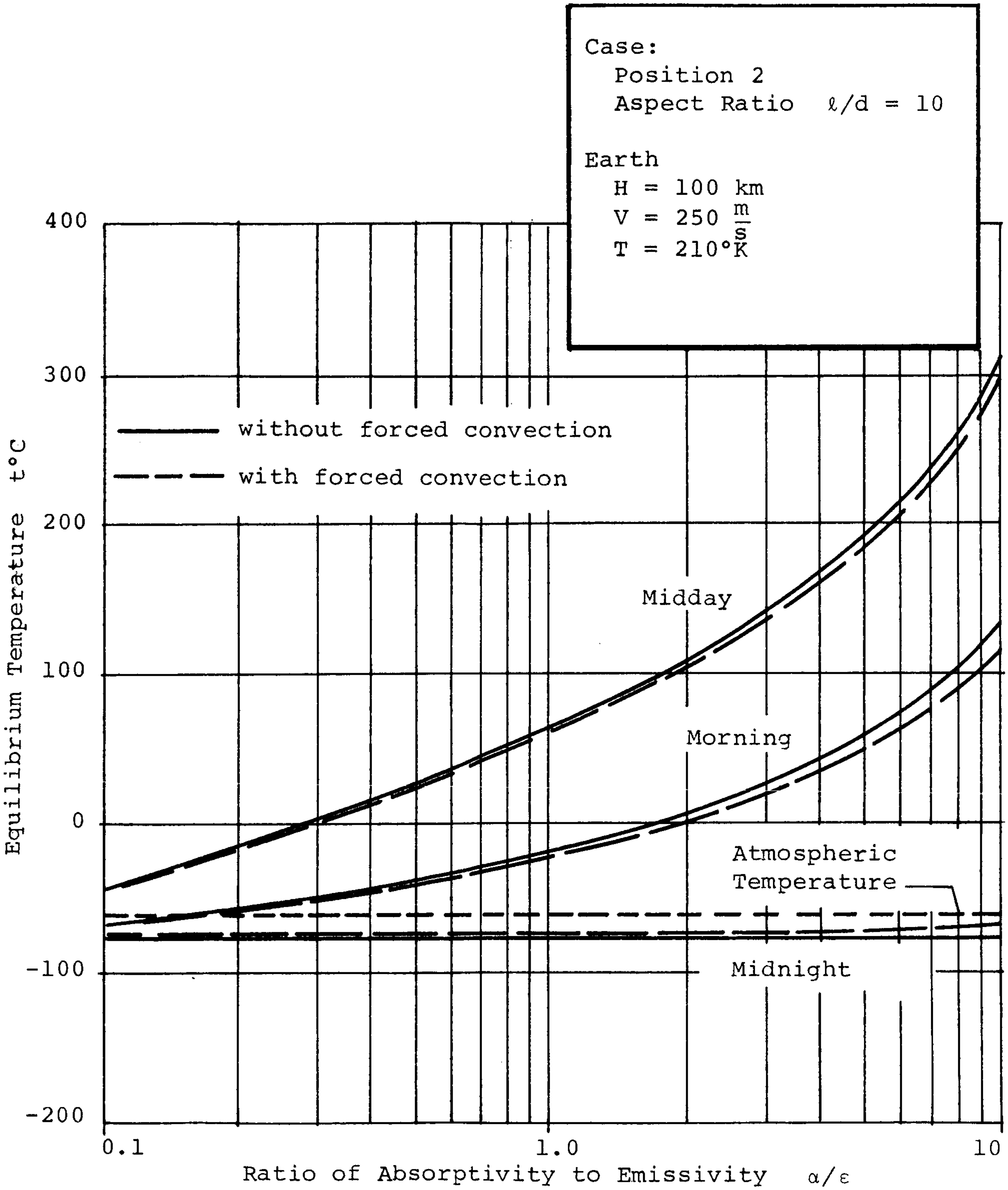


Figure 4. Equilibrium Temperatures for an Inflated Column: Variation of Daytime and Influence of Forced Convection Term

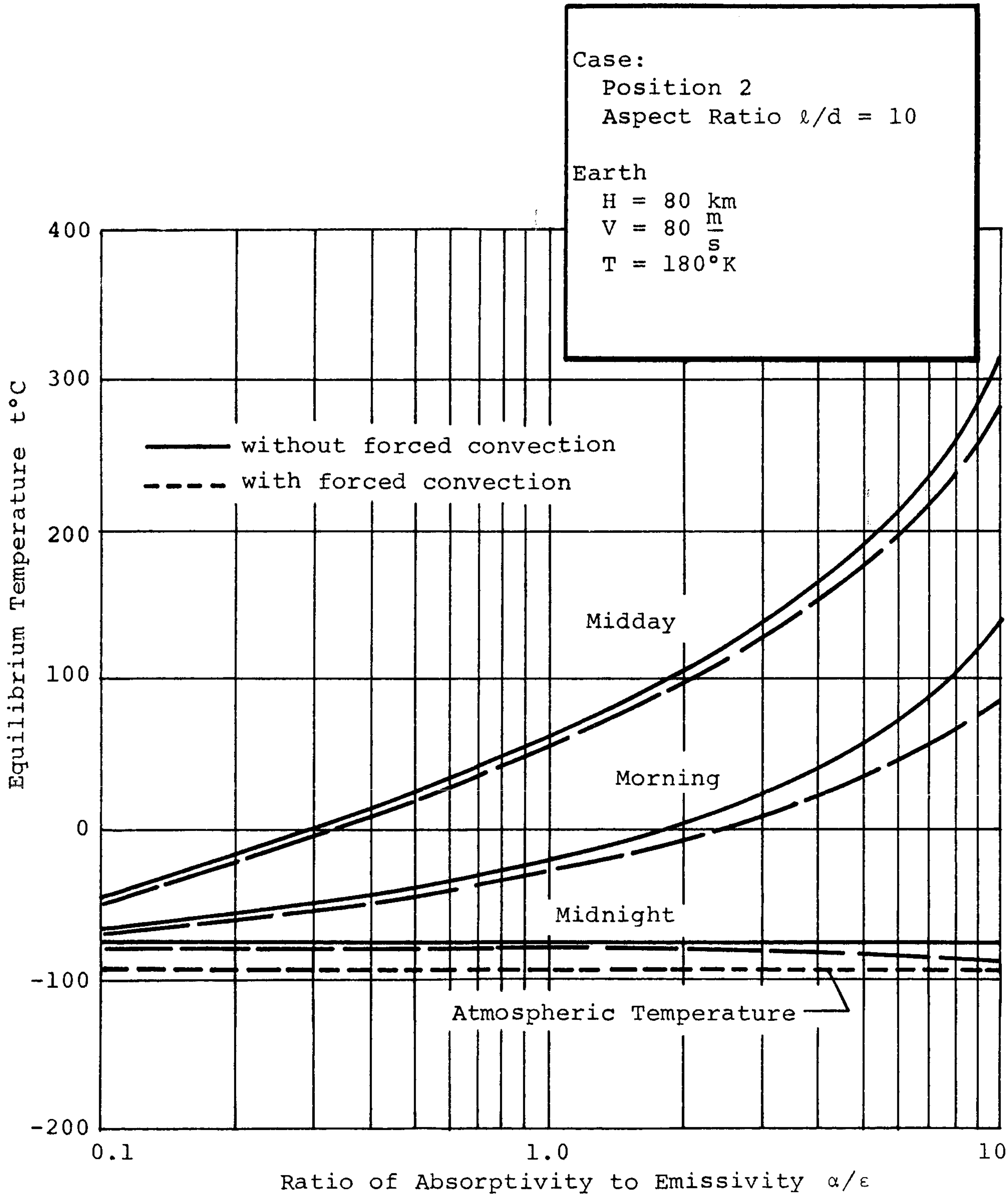


Figure 5. Equilibrium Temperatures for an Inflated Column: Variation of Daytime and Influence of Forced Convection Term

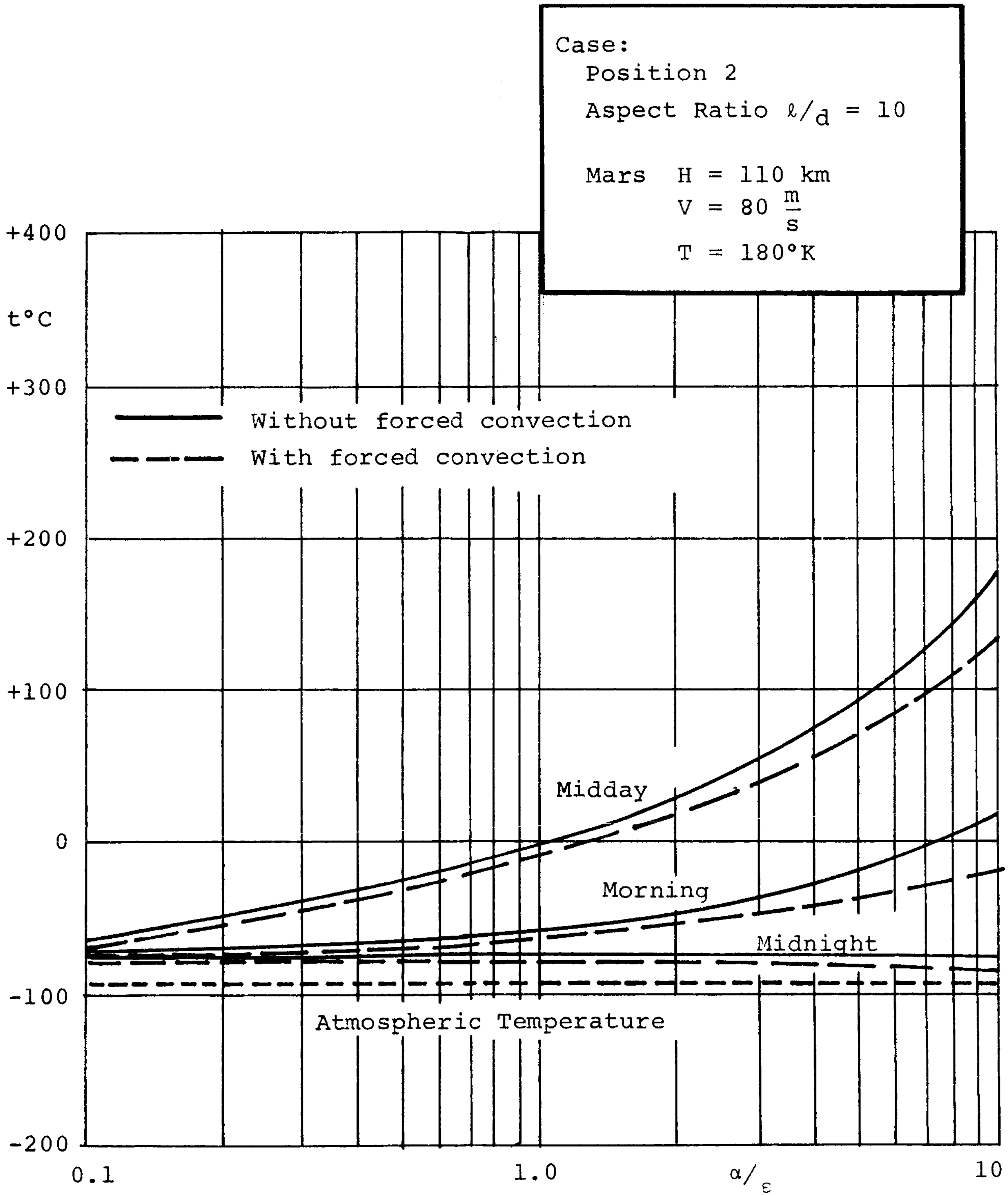


Figure 6. Equilibrium Temperatures for an Inflated Column: Variation of Daytime and Influence of Forced Convection Term

THE PRESSURIZATION OF AN INFLATED STRUCTURE

Packing Volume Ratio

The inflation pressure that can be obtained depends primarily on the amount of gas used to inflate a given volume at a given temperature. The amount of gas used can conveniently be described by a packing volume ratio, i.e., by the ratio between the compressed volume V_p (at earth surface conditions) and the fully deployed volume V_d :

$$v = \frac{V_p}{V_d} \tag{4}$$

Three kinds of pressurization media will be investigated:

1. A gas, which expands for beam deployment; example: air
2. A liquid, which vaporizes for deployment action; examples: freon 11 (trichlorofluoromethane), methanol, liquid nitrogen
3. A liquid-gas mixture; example: freon 11-air, methanol-air.

Basic assumptions:

- the substance is packaged at launching site atmospheric conditions: 760 mmHg/20°C (except for liquid nitrogen at - 196°C): subscript "p"
- it is fully gaseous after deployment: subscript "d"

Assume two substances, characterized by subscripts 1 and 2. Then

$$\left.\begin{aligned} p_{d_1} V_d &= m_1 R_1 T_d \\ p_{d_2} V_d &= m_2 R_2 T_d \end{aligned}\right\} \tag{5}$$

With p_{d_1} and p_{d_2} the partial pressures of the component, the total pressure of the mixture is

$$\bar{p}_d = p_{d_1} + p_{d_2} = \bar{\rho}_d \bar{R} T_d \tag{6}$$

The gas constant of the mixture can be expressed as

$$\bar{R} = \frac{R_1 m_1 + R_2 m_2}{m_1 + m_2} \tag{7}$$

or with the introduction of the total mass

$$m = m_1 + m_2$$

We define:

$$m_1 = am$$

$$m_2 = (1 - a)m$$

$$\bar{R} = R_1 a + R_2 (1 - a) \tag{8}$$

Taking the volumes:

$$V_d = \frac{m}{\bar{\rho}_d}$$

$$V_p = \frac{am}{\rho_{p_1}} + \frac{(1 - a)m}{\rho_{p_2}}$$

we get combined with equation (6)

$$\overline{p_d} = v \cdot \frac{\bar{R} \cdot T_d \rho_{p_1} \cdot \rho_{p_2}}{a\rho_{p_2} + (1 - a)\rho_{p_1}} \tag{9}$$

If only one component is present, equation (9) simplifies to

$$p_d = v \cdot R \cdot T_d \rho_p \tag{10}$$

For specific cases with mixtures, it has to be verified that the partial pressure of the liquid component is below its saturation pressure!

Choice of Pressurization Substances

Figure 7 shows vapor pressure curves for some substances which are liquid under normal conditions at the earth's surface. It can be seen that freon 11 has the lowest saturation line available. Its low gas constant makes it a relatively heavy gas after evaporation. Methanol has a gas constant similar to that for air, it will be seen, however, that its higher vapor pressure line limits the range of its applicability.

Table 2: Pressurization Substances

Substance	$R\left[\frac{J}{°K kg}\right]$	$\rho\left[\frac{kg}{m^3}\right]$
Air	287	1.20 *
Liquid Nitrogen	296.8	$0.804\ 10^3$ **
Freon 11	60.5	$1.494\ 10^3$ *
Methanol	259.5	$0.793\ 10^3$ *

*) at 760mmHg 20°C
**) at 760mmHg - 196°C

Figures 8 and 9 show pressure curves versus temperature for the investigated pure substances in their deployed condition. They show the following two most important features:

1. Variation of pressure with temperature is of minor importance as long as the pressurization substance is in a purely gaseous phase (a factor 2 approximately between - 100°C and + 100°C).

2. Along the saturation line, however, the pressure varies strongly with temperature (about 4 orders of magnitude between - 100°C and + 100°C).

Conclusion

Any practical design of a pressurized beam should avoid the saturation line:

An error in temperature higher than expected might result in pressures too high to be contained by the skin. If the temperature is lower than expected the beam may not fully inflate.

Pressure curves for the mixtures freon-air and methanol-air are given on figure 10 for a temperature 0°C . Results in figures 8 to 10 are summarized in figure 11 which show the range of applicability of the various pressurization substances in terms of pressure and packing volume ratio. For any deployment pressure required and for any packing volume ratio available, a suitable pressurizing substance can now be chosen for a given operating temperature. Low packing volume ratios call for liquids, the most effective of which is liquid nitrogen. Mixtures with air allow for any selection of packing volume provided enough space is available in the transportation container.

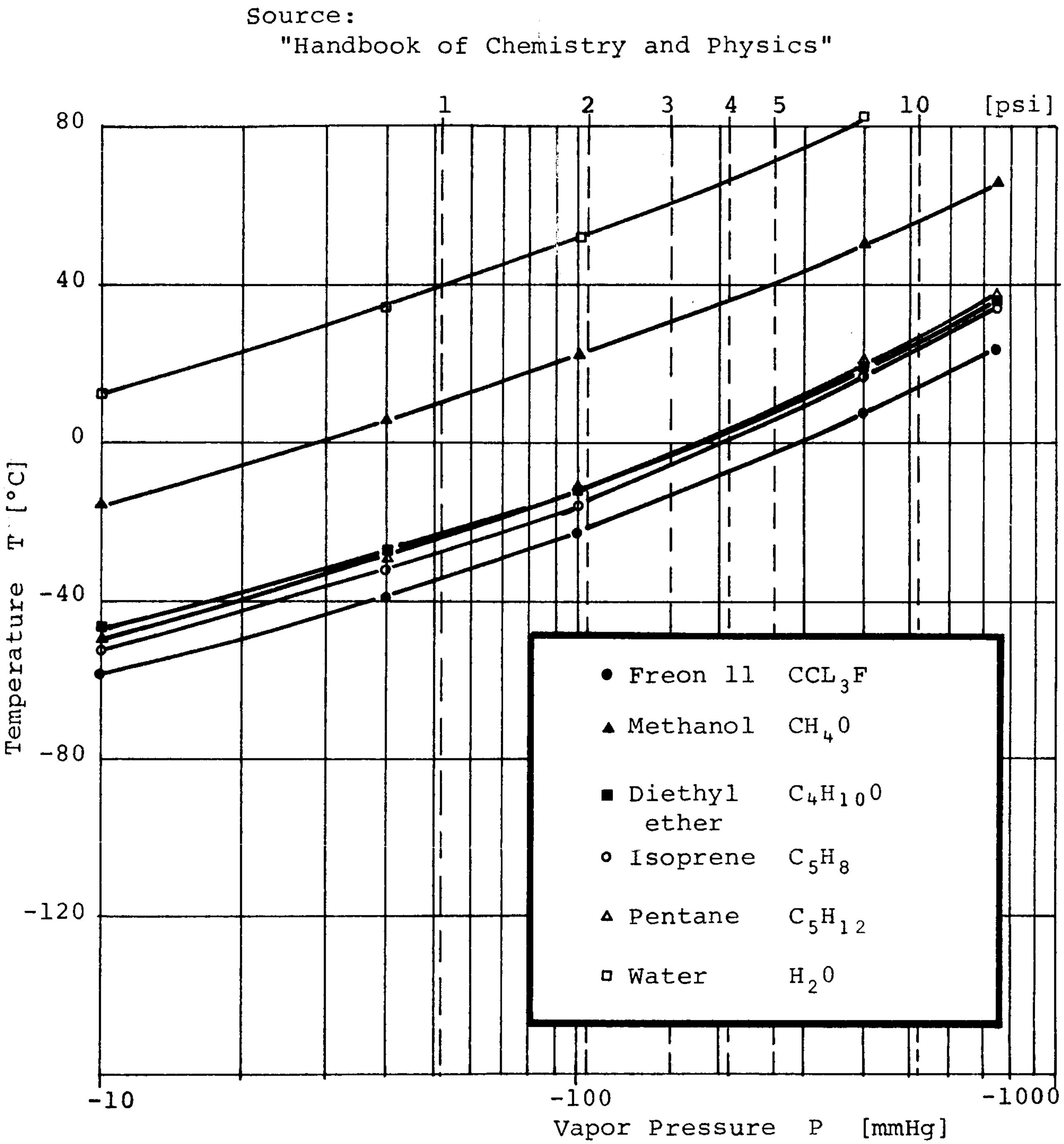


Figure 7. Vapor Pressure Curves for Some Substances Suitable to Pressurize an Inflated Column

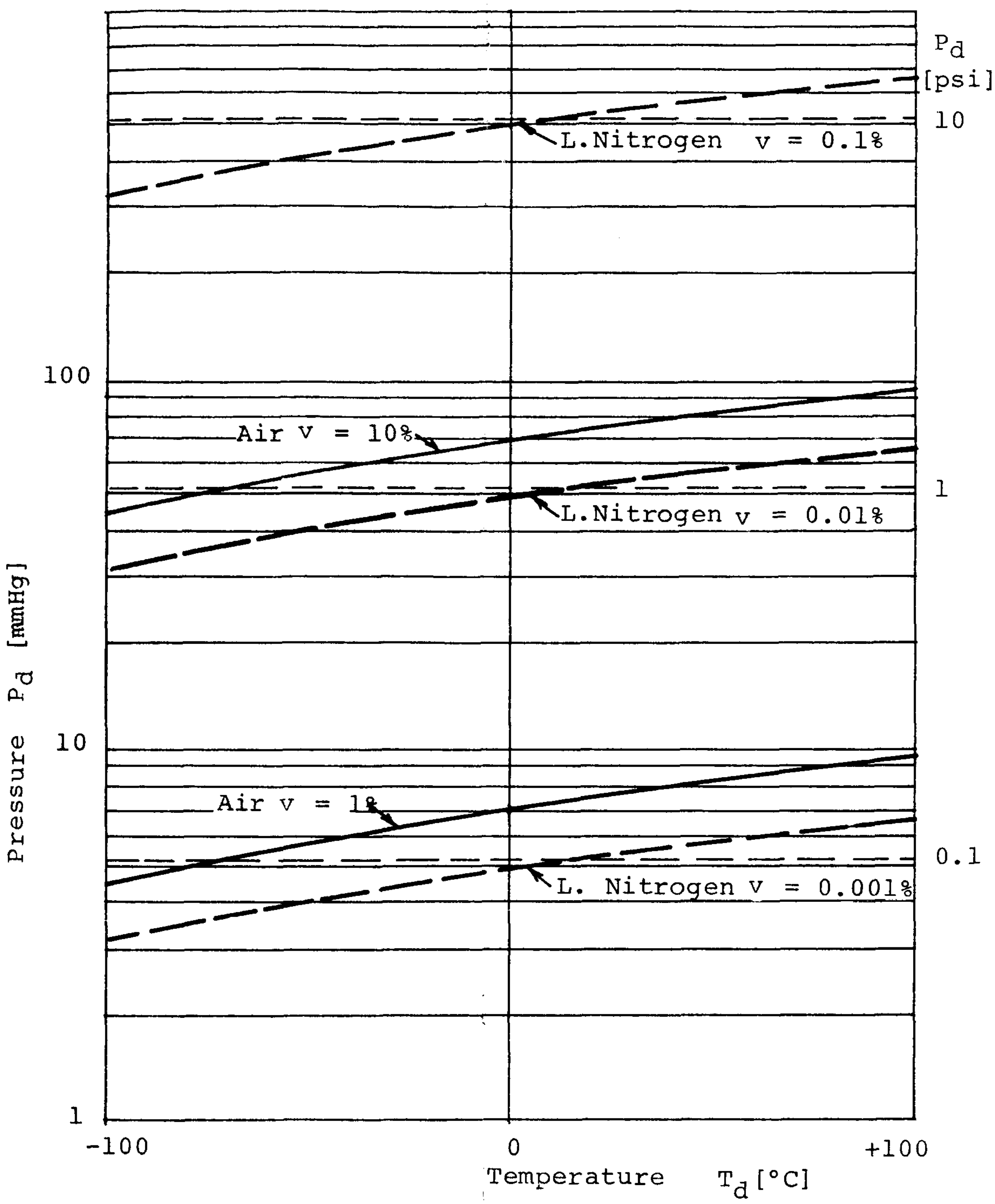


Figure 8. Pressure Curves for Deployed Column Using Air or Liquid Nitrogen as Pressurization Medium

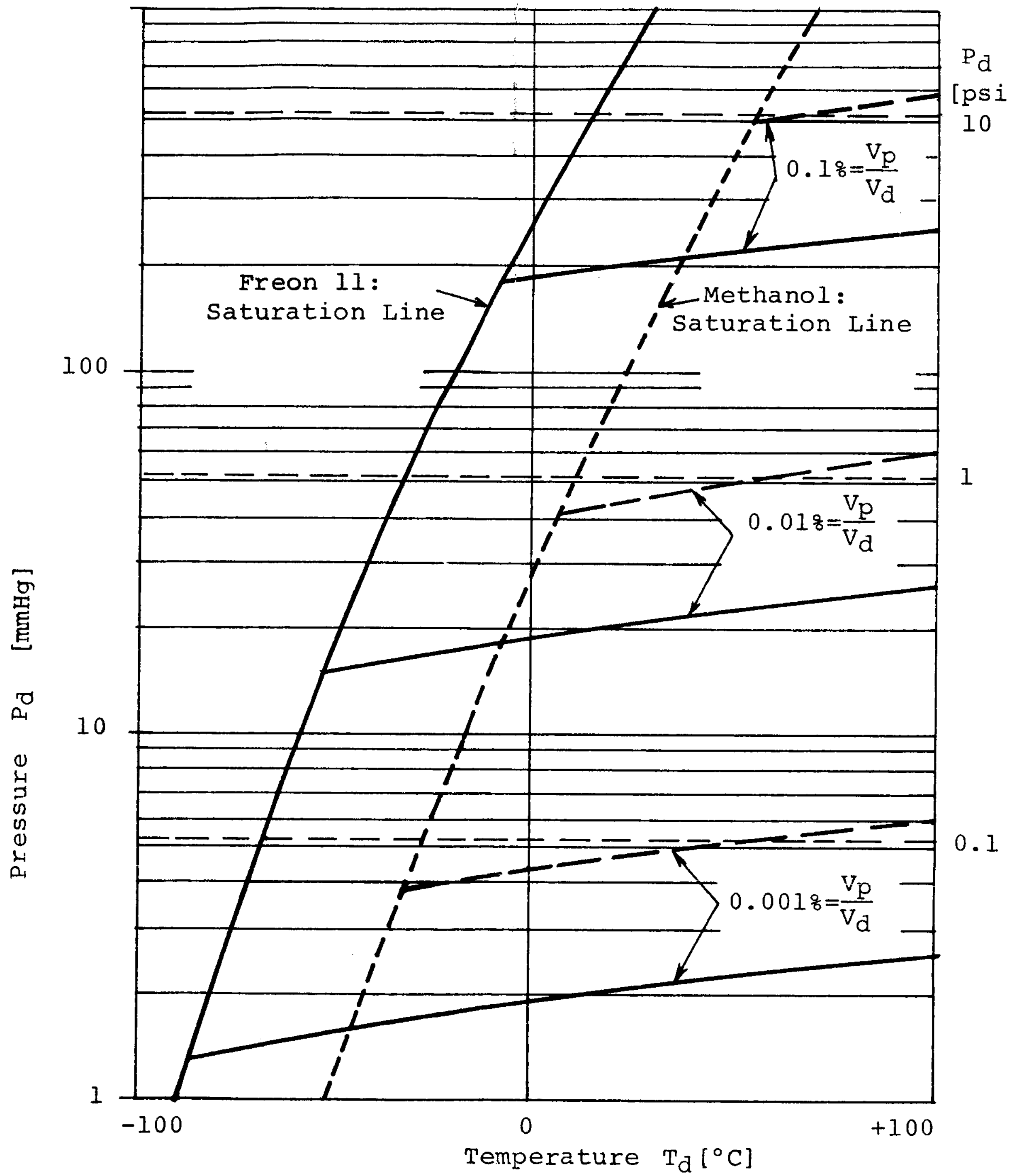


Figure 9. Pressure Curves for Deployed Column Using Freon 11 or Methanol as Pressurization Medium

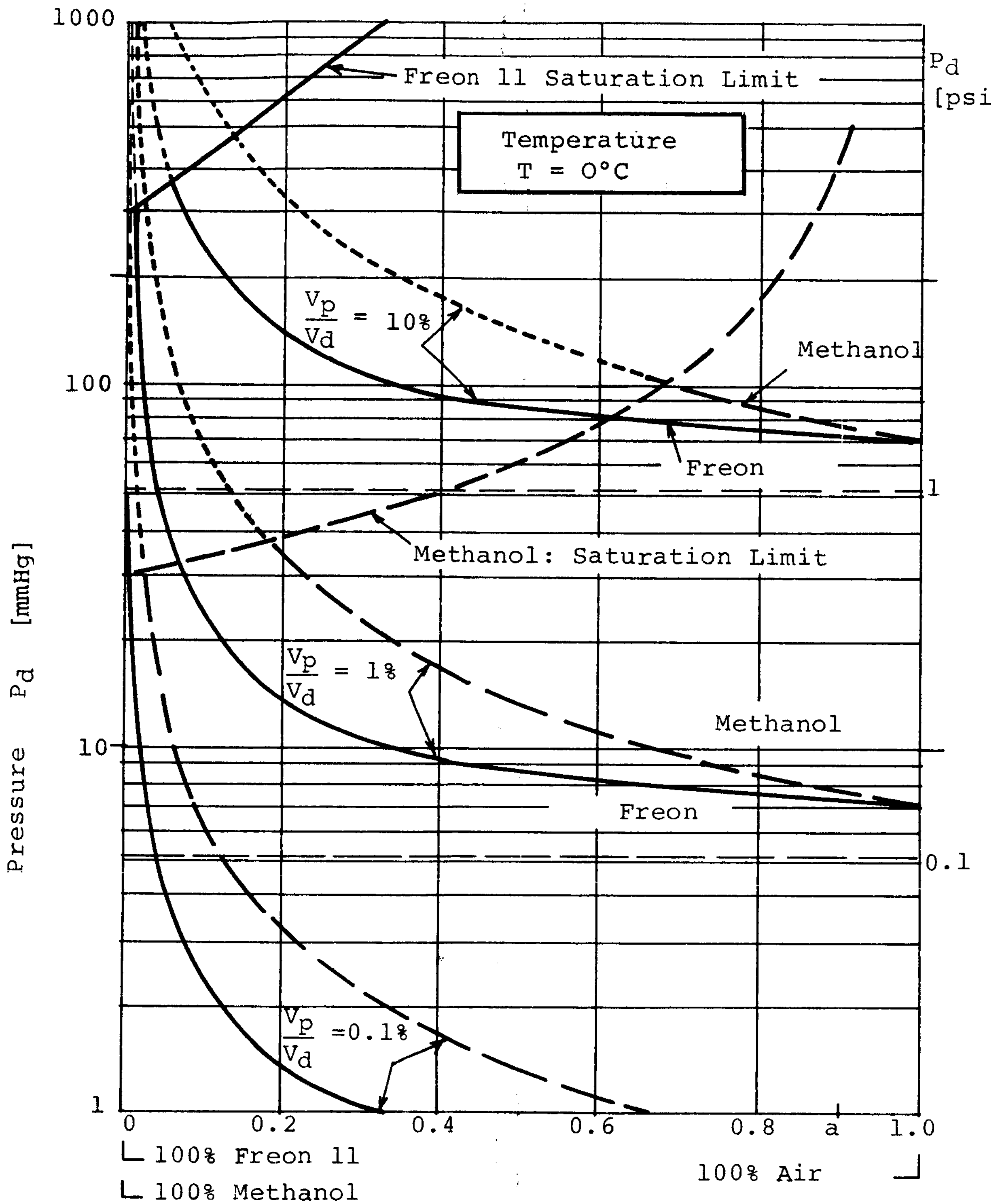


Figure 10. Pressure Curves for Deployed Columns Using a Methanol - Air Mixture or a Freon 11 - Air Mixture as Pressurization Medium. Temperature T = 0°C

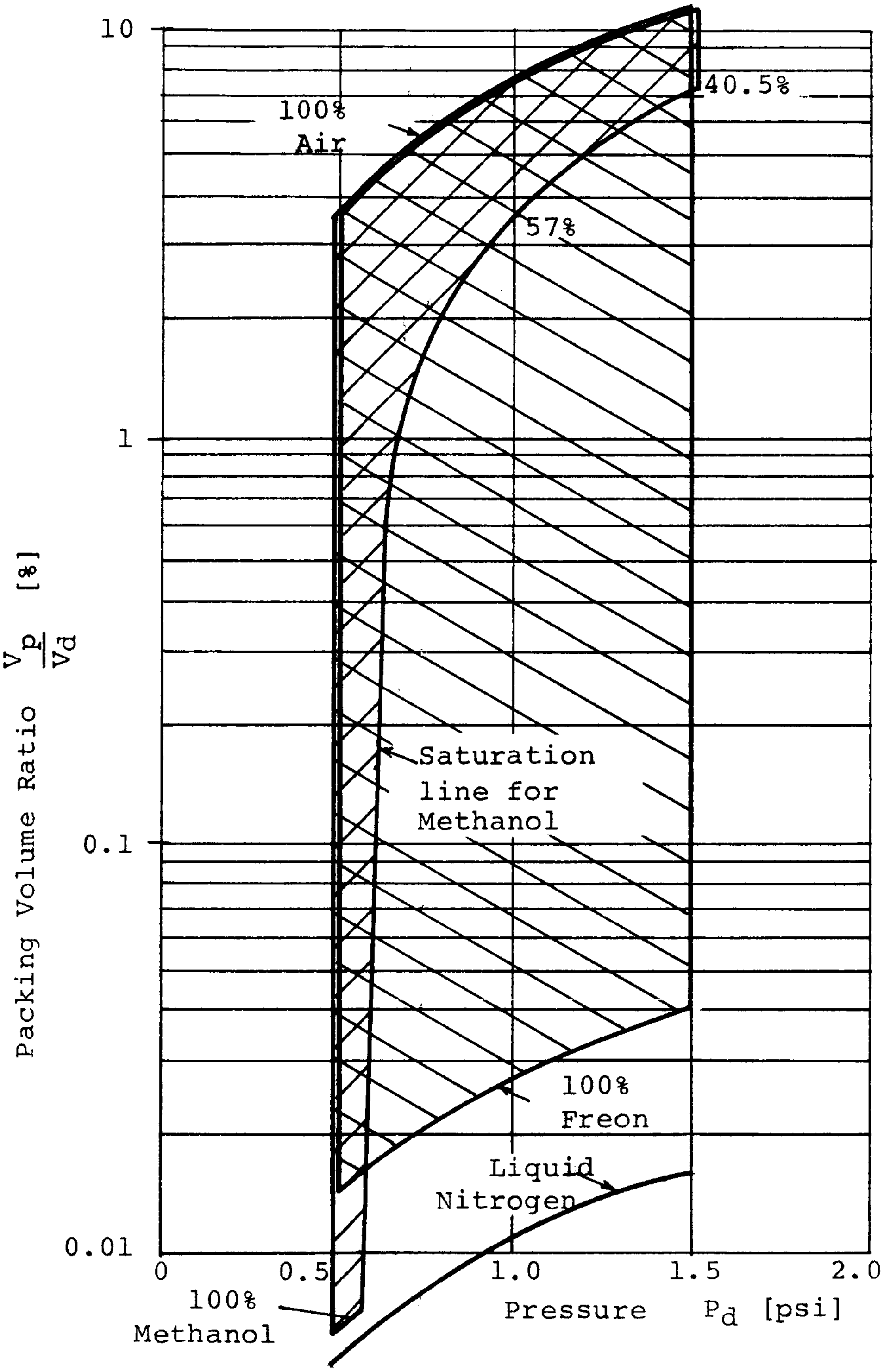


Figure 11. Range of Applicability of Various Pressurization Media for a Temperature $T = 0°C$

MINIMUM WEIGHT DESIGN OF INFLATED COLUMNS

A thin walled cylindrical column of radius r, length ℓ and wall thickness Δ is considered. It is to support a compressive load P at its ends. The column is inflated to a pressure p.

The Euler buckling formula for a pin ended column states that

$$P = \frac{\pi^2 EI}{\ell^2} \quad \text{where} \quad I = \pi\Delta r^3 \tag{11}$$

is the maximum load without buckling. P can always be expressed in terms of the internal column pressure

$$P = k \cdot r^2 \pi p \tag{12}$$

k is a factor ranging from 0 to 1 ; and $k = 1$ yields the maximum compressive load without compressive axial stresses in the wall. The hoop stress for a thin walled tube is

$$\sigma = \frac{rp}{\Delta} \tag{13}$$

and σ is constrained by

$$\sigma \leq \sigma_{max}$$

for a given skin material.

Eliminating p and r from equations (11), (12), and (13) yields

$$\sqrt[3]{\frac{P\ell^2}{\pi^3 E\Delta}} = \frac{P}{k\pi\Delta\sigma}$$

or

$$k^3 = \frac{P^2}{\ell^2}\left[\frac{E}{\sigma^3 \Delta^2}\right] \qquad (14)$$

where (P^2/ℓ^2) is given by the operational requirements and $(E/\sigma^3 \Delta^2)$ is given by the chosen material. Combining equations (11), (12), and (13), but eliminating P , p and Δ we get:

$$AR = \frac{\ell}{r} = \pi \sqrt{\frac{E}{\sigma}} \cdot \sqrt{\frac{1}{k}} \qquad (15)$$

The value

$$AR_1 = \pi \sqrt{\frac{E}{\sigma_{max}}}$$

with the maximum $k = 1$ is the minimum aspect ratio ever obtainable. It is a pure material constant. An AR smaller than AR_1 would always introduce compressive axial stresses into the membrane, or load it beyond the maximum allowable stress limit.

The weight of the cylindrical portion of the column membrane and of the pressurization gas is

$$W = g\ell\pi\left(2r\Delta\rho_s + r^2\rho_g\right) \qquad (16)$$

Equation (16) can be expressed in terms of the operational requirements P and ℓ as follows

$$W = \left(\frac{P}{k}\cdot\ell\right) g \left(2\cdot\frac{\rho_s}{\sigma} + \frac{1}{RT}\right) \qquad (17)$$

or with equation (14):

$$W = \sqrt[3]{P\ell^5}\; g\, \sqrt[3]{\frac{\Delta^2}{E}} \left(2\rho_s + \frac{\sigma}{RT}\right) \tag{18}$$

Equation (17) suggests a minimum weight for a column designed for maximum hoop stress σ_{max}. Equation (18) however apparently claims the contrary and suggests a column designed with the minimum available wall thickness Δ_{min}. Selecting a ficticious characteristic number k^* based upon equation (14):

$$k^{*3} = \frac{P^2}{\ell^2} \left[\frac{E}{\sigma_{max}{}^3 \cdot \Delta_{min}{}^2}\right]$$

For $k^* > 1$, minimum weight is obtained by a maximum stress design, at $k = 1$, with $\Delta > \Delta_{min}$.

For $k^* < 1$, minimum weight is obtained by a minimum thickness design, at $k = 1$, with $\sigma < \sigma_{max}$.

This feature is exemplified in Figure 12.

Assume a sample case with

$$P = 1 \text{ N} \quad \text{and} \quad \ell = 1 \text{ m}$$

and a pressurization medium Freon 11 at 0°C. The membrane is assumed to be a Mylar foil with a density

$$\rho_s = 1.38 \cdot 10^3 \frac{\text{kg}}{\text{m}^3}$$

and a modulus of elasticity

$$E = 3.795 \cdot 10^9 \frac{\text{N}}{\text{m}^2} \; (5.5 \cdot 10^5 \text{ psi})$$

Mylar has a maximum stress permissible

$$\sigma_{max} = 3.45 \cdot 10^7 \frac{\text{N}}{\text{m}^2} \; (5.10^3 \text{ psi})$$

(or a $\varepsilon = \frac{\sigma_{max}}{E} \simeq 1\%$).

The thinnest available Mylar foil is 0.25 mil Mylar with

$$\Delta_{min} = 0.635 \cdot 10^{-5} \text{ m}$$

Since

$$k^{*3} = 0.23 \cdot 10^{-2} < 1 \qquad \text{(Point P on figure 12)}$$

the minimum weight is obtained with a design for minimum thickness and at $K = 1$:

$$W_{min} = 0.00627 \text{ N}$$

The hoop stress is then far below its tolerable maximum and an inflation pressure higher than anticipated would not burst the membrane. Figure 12 shows another advantage of a column designed for minimum thickness. A safety factor of 2 (or k = 0.5) against the critical compression load for local buckling requires less than 10% additional weight over the minimum at $k = 1$. Figure 13 shows that the minimum packing volume ratio coincides with the minimum weight. With Equation (10) the packing volume can be expressed as

$$v = \frac{p}{RT\rho_p} \left(\frac{\sigma\Delta}{r}\right) \left(\frac{1}{RT\rho_p}\right)$$

The radius r is given by

$$r = \frac{1}{\pi} \sqrt[6]{P^2 \ell^4 \frac{1}{\Delta^2 E^2}}$$

The minimum weight can further be influenced by the proper choice of the pressurization medium with a gas constant as high as possible. This feature is, however, of minor importance. At the point of minimum weight (Figure 12) there is

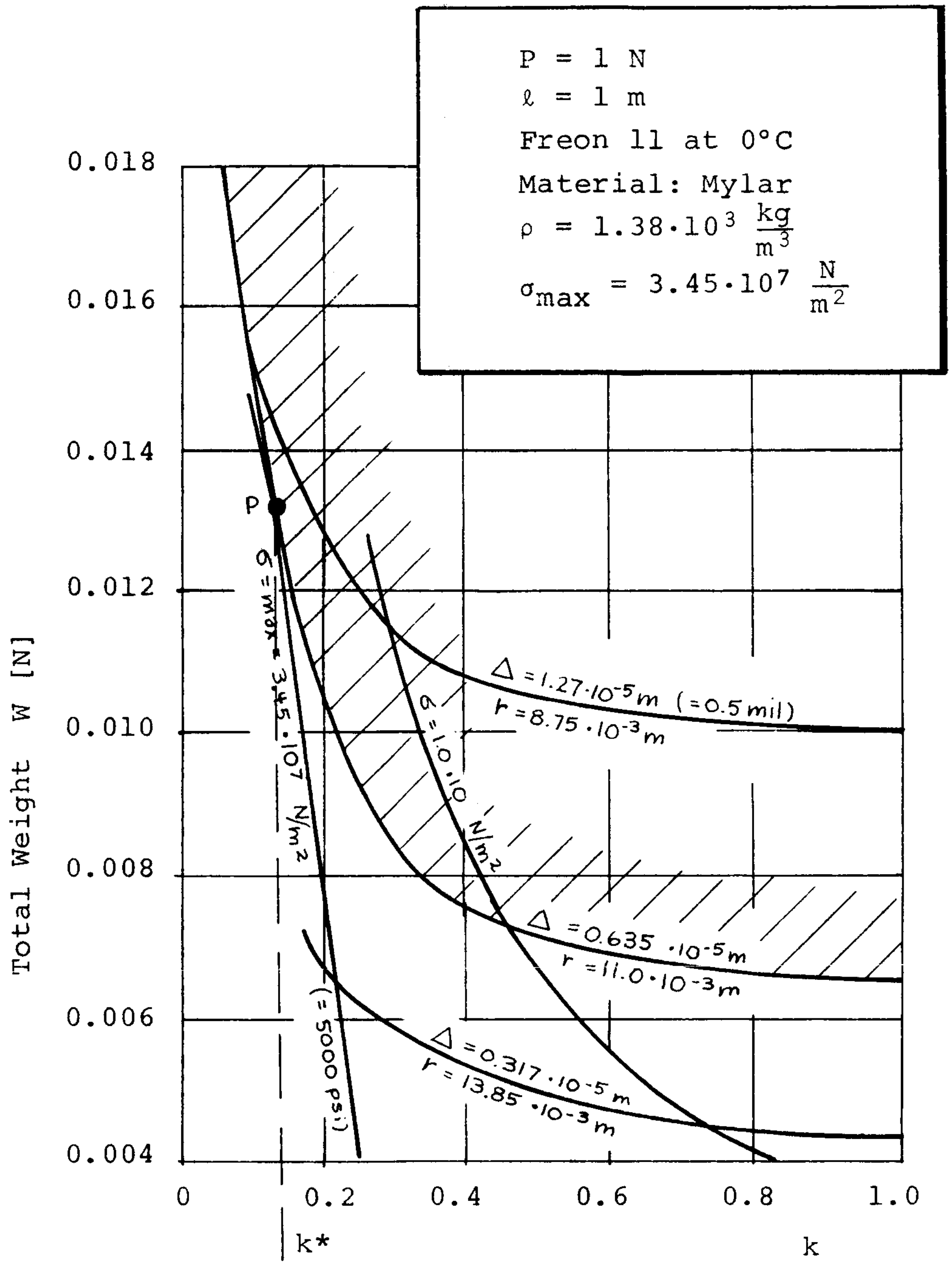


Figure 12: The Minimum Weight of a Pressurized Column
The shaded area is the region of practical design of the column.
($\Delta_{min} = 0.635 \cdot 10^{-5}$ m)

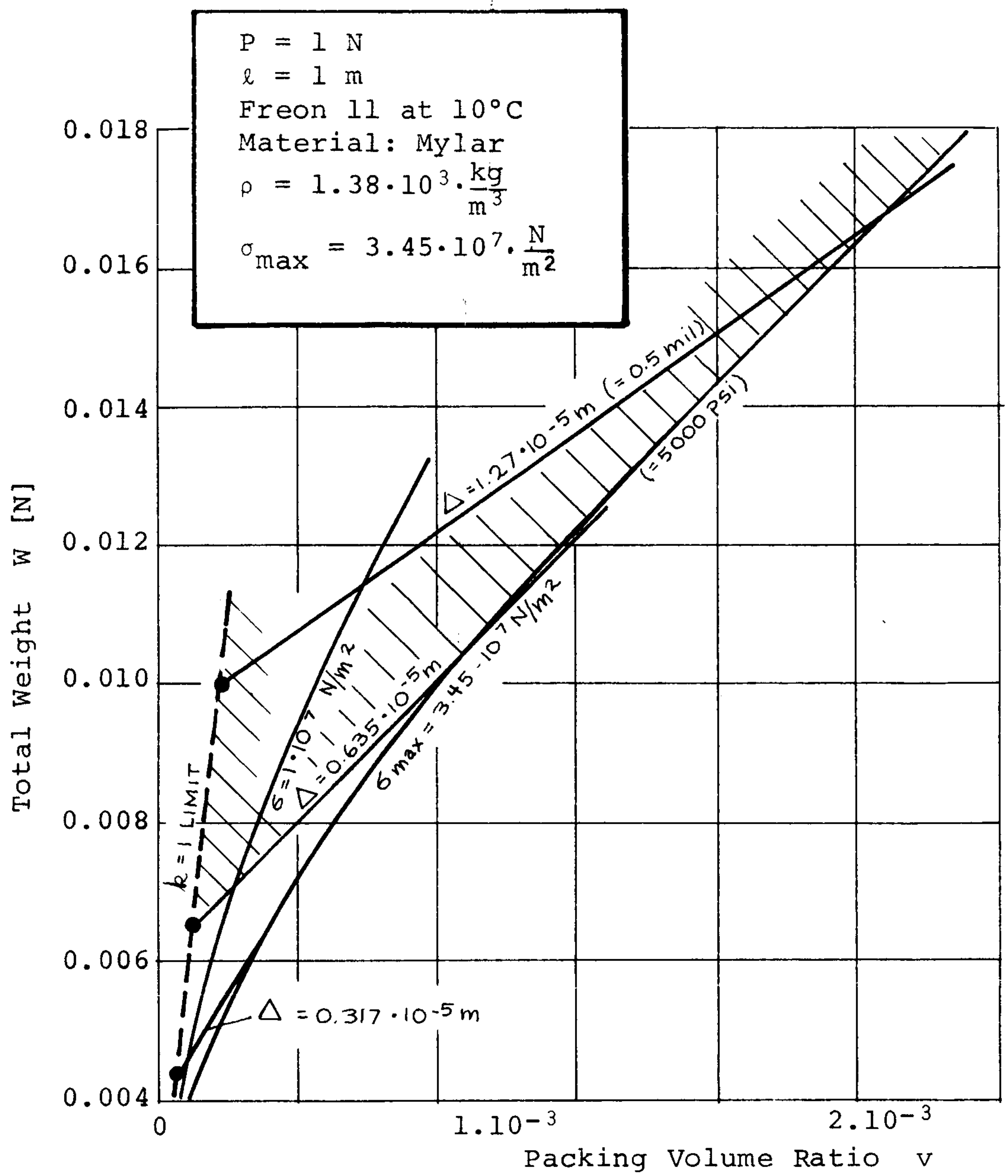


Figure 13: The Minimum Packing Volume Ratio of a Pressurized Column
The shaded area is the region of practical design of the column ($\Delta_{min} = 0.635 \cdot 10^{-5}$ m)

$$2\rho_S = 2.76 \cdot 10^3 \ \frac{kg}{m^3}$$

$$\frac{\sigma}{RT} = 0.275 \cdot 10^3 \ \frac{kg}{m^3}$$

If the column is designed for maximum stress the term σ/RT might well be of the same order of magnitude as $2\rho_S$.

CONCLUSIONS

Design criteria are established for the proper inflation of thin walled structures in a near planetary spacial environment. Equilibrium temperature calculations for two earth altitudes and one for Mars show that the heating up of a structure primarily depends on the ratio of solar absorptivity to emissivity, α/ε , of its surface, sun radiation being the most important heat source. Heat transfer due to forced convection, created by the body moving at subsonic speed through the atmosphere, is noticable but generally small compared to direct solar radiation. Temperature extremes range from +300°C (mid-day, earth, high α/ε) down to temperatures close to the local atmospheric value -80°C generally reached by the midnight cases.

The numerical example in Appendix E of the design of the inflated X-braces of a high altitude decelerator illustrates the theory presented in this report.

The choice of the pressurizing medium depends largely on the available packaging volume. The use of residual air at atmospheric pressure is limited to very high packing volume ratios of approximately 10%. Lower packing volume ratios of approximately 0.01% are possible by using vaporizing liquids or liquid-air mixtures. To avoid accidental over - or under pressurization the saturation limits of the pressurizing liquid should be avoided. Among the materials considered, Freon 11 offers the widest range of applicability. Due to its low gas constant a Freon 11 - pressurized column is however expected somewhat heavier than methanol and similar liquids.

Weight considerations show that the lightest columns are obtained with the thinnest skin. It can be shown that the column is generally lighter when designed for minimum skin thickness than for maximum hoop stress.

REFERENCES

1. "Handbook of Astronautical Engineering", H. H. Koelle, McGraw Hill, Section 22, page 104 ff.

2. "Handbook of Chemistry and Physics", The Chemical Rubber Publishing Co.

3. "U. S. Standard Atmosphere 1962".

4. "NASA Engineering Models of the Mars Atmosphere for Entry Vehicle Design", George M. Lewis et al, NASA TN D-2525, 1964.

5. "Heat and Mass Transfer", E.R.G. Eckert and R. M. Drake, McGraw Hill.

6. "Structural Design of Missile and Spacecraft", L. H. Abraham, McGraw Hill.

7. "Basic Studies on the Use and Control of Solar Energy", Edwards et al, UCLA Report 60-93, part V.

8. "Development of a High Altitude Stokes Flow Decelerator", P. Niederer, ARC-R-236.

Astro Research Corporation
Santa Barbara, California, February 16, 1967.

APPENDIX A

ESTIMATION OF HEATING RATES

The question arises in what time a column heats up to the estimated temperature equilibrium. As initial conditions it is assumed that the column and its pressurizing gas have the same temperature as the surrounding atmosphere. The forced convection term of Equation (1) is neglected for this estimation. A further simplification is introduced with the assumption that the whole structure and its pressurization gas are always isothermal. Then

$$\frac{dT_s}{dt} = \frac{dT_g}{dt}$$

Equations (1) and (2) combined give then

$$T_{s_o}{}^4 - T_s{}^4 = \left[\frac{\rho_s \; C_{p_s} \; \Delta}{\varepsilon \; \sigma} + \frac{\rho_g \; C_{p_g} \; V}{\varepsilon \; \sigma \; A}\right] \cdot \frac{dT_s}{dt}$$

T_s being a function with time between $T_s = T_a$ at $t = o$ up to $T_s \rightarrow T_{s_o}$.

Numerical example:

Case: "Earth 100 km", midday, position 2

$AR = 200, \; \frac{\alpha}{\varepsilon} = 10 \quad \varepsilon = 0.1$

$T_a \cong 210 \; [°K] \qquad T_{s_o} = 587 \; [°K]$

Skin: 0.5 mil Mylar $\Delta = 1.270 \cdot 10^{-5}$ [m]

$\rho_s = 1.38 \cdot 10^3 \left[\frac{N}{m^2}\right]$

$C_{p_s} \cong 0.25 \left[\frac{kcal}{k_g °K}\right]$

Gas: Freon 11

$$C_{p_g} = 0.115 \left[\frac{kcal}{kg\,°K}\right] \qquad R_g = 60.5 \left[\frac{J}{kg\,°K}\right]$$

$\frac{V}{A} \approx \frac{r}{2} = 0.025$ [m] if the reference tube diameter $d = 0.1$m is used again. According to Part III a maximum internal pressure

$$p_g = \frac{\sigma\Delta}{r} = 87.6\cdot 10^2 \left[\frac{N}{m^2}\right] \quad (= 1.27 \text{ psi})$$

can be allowed, with a maximum working stress for the Mylar foil:

$$\sigma = 3.45\cdot 10^7 \left[\frac{N}{m^2}\right] \quad (= 5.10^3 \text{ psi})$$

Assuming a fully deployed volume, which is rapidly reached after exposure to the radiation, the density is

$$\rho_g = \frac{p_g}{R_g\, T_{s_o}} = 0.2465 \left[\frac{kg}{m^3}\right]$$

With these values, Equation (4) gets

$$\frac{dT_s}{dt} = 1.155\cdot 10^5 - T_s{}^4\cdot 0.973\cdot 10^{-6}$$

This equation is graphically shown in Figure A1. The equilibrium temperature T_{s_o} is reached to 98% after only 21 seconds

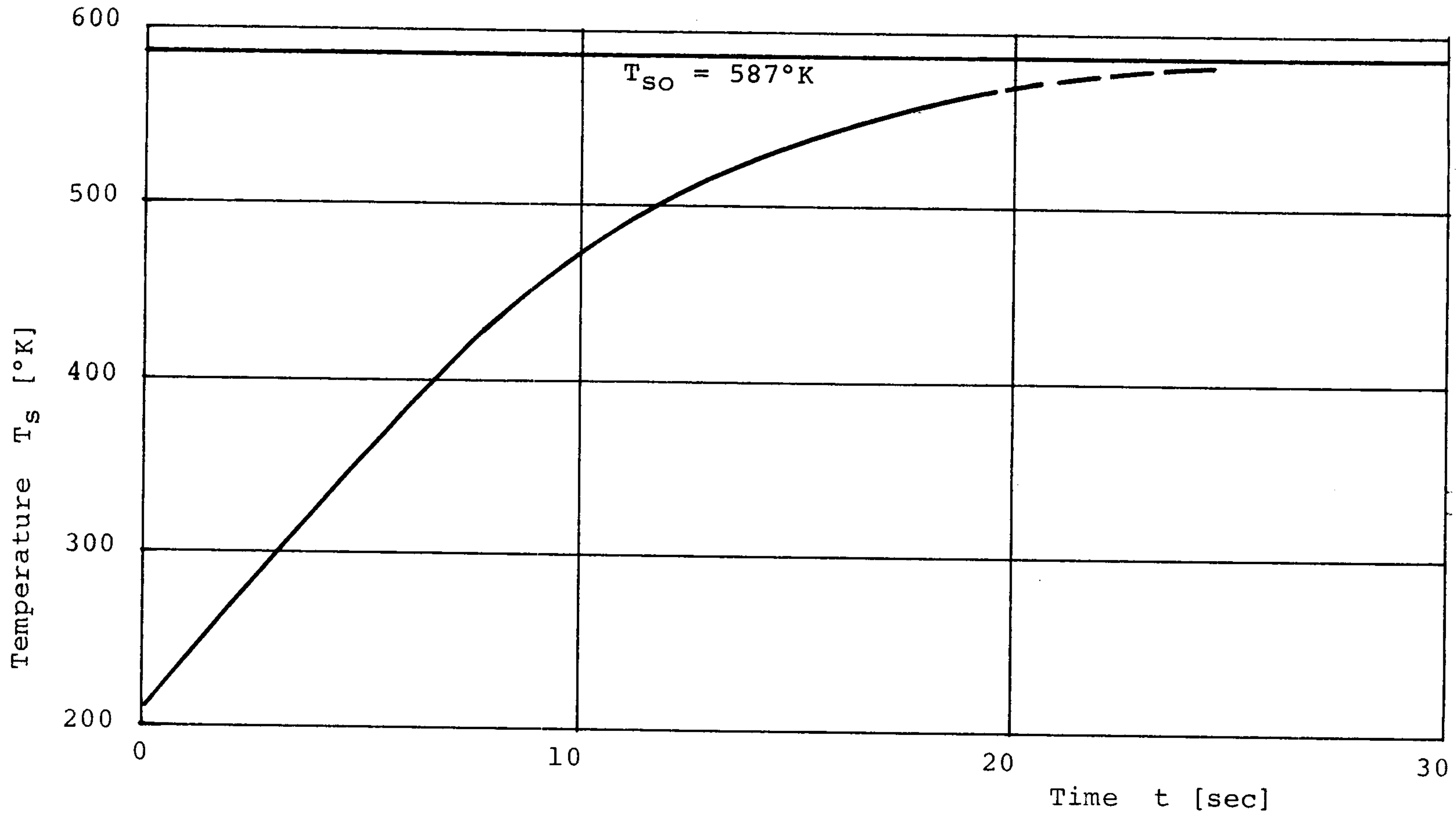
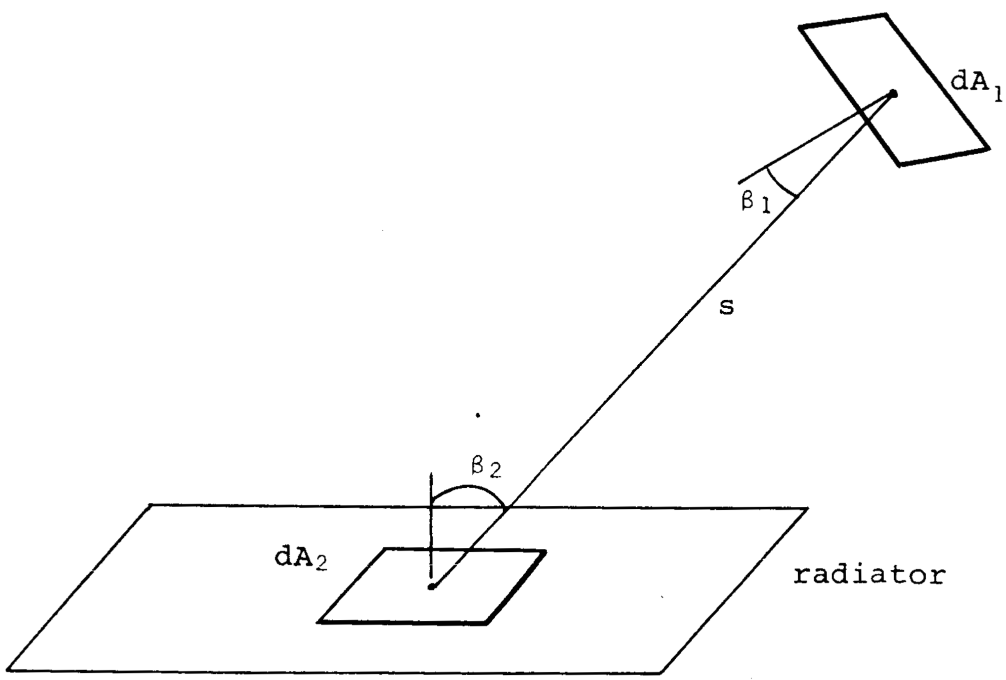


Figure A-1. Heating Rate Estimation: Earth 100 km, Midday, Position 2, $\alpha/\varepsilon = 10$

APPENDIX B

DETERMINATION OF THE RADIATIVE HEAT TRANSFER SHAPE FACTORS AND OF THE COMPOSITE AREAS A_S AND A_B

The general equation for the heat exchange between two small black surfaces is



$$d^2Q = \frac{\cos\beta_1 \cdot \cos\beta_2}{S^2}\, dA_1\, dA_2\, \frac{\sigma}{\pi}\,(T_1^{\,4} - T_2^{\,4}) \qquad \text{(B-1)}$$

If it is assumed that dA_2 is an area element of the radiator, then all the geometrical relations between dA_1 and dA_2 can be put into a shape factor dF of dA_1 with respect to dA_2:

$$dF = \frac{\cos\beta_1 \cos\beta_2}{S^2\pi} dA_2 \qquad \text{(B-2)}$$

and Equation (B-1) becomes

$$d^2Q = \sigma \, dF \, dA_1 \, (T_1{}^4 - T_2{}^4) \qquad \text{(B-3)}$$

Integrating dF over the whole radiator we get

$$dQ = \sigma F \, dA_1 \, (T_1{}^4 - T_2{}^4) \qquad \text{(B-4)}$$

$$F = \int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi S^2} dA_2 \qquad \text{(B-5)}$$

For the present purpose the following assumptions are made for the radiator:

1. The overall size of the beam is negligible compared with the size of the radiating surface.

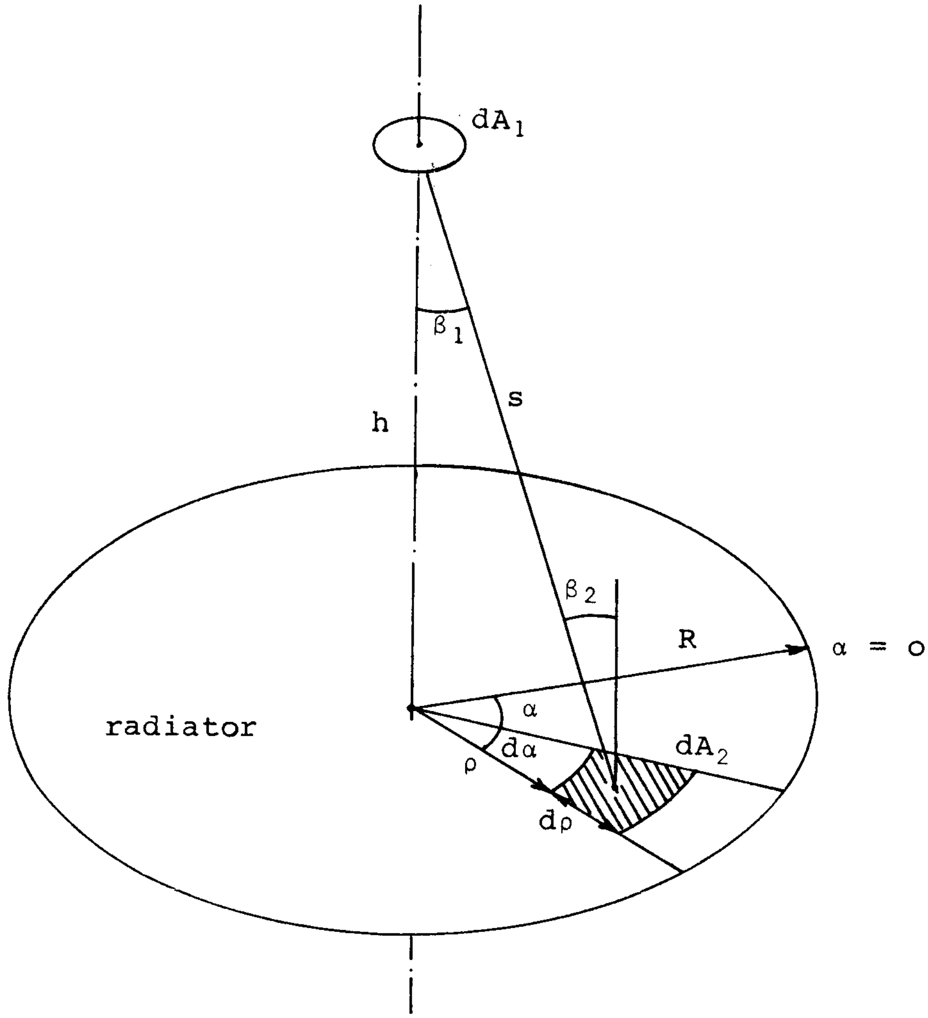
2. The radiating surface (planet) is assumed to be flat and circular in shape.

3. Its size is such that it occupies the same spatial angle as the planet when viewed from dA_1 .

4. The position of dA_1 is over the center of the radiating disc.

SHAPE **FACTOR** F_1 (A_1 parallel radiator)

It is assumed that dA_1 is parallel dA_2 and h above it:



$$dA_2 = \rho d\alpha d\rho$$

$$s^2 = \rho^2 + h^2$$

$$\cos \beta_1 = \frac{h}{s} \qquad \cos \beta_2 = \frac{h}{s}$$

$$F_1 = \frac{1}{\pi}\int_0^R \int_0^{2\pi} \frac{h^2}{S^2 \ (\rho^2 + h^2)} \rho d\alpha d\rho$$

$$= 2\int_0^R \frac{h^2}{(\rho^2 + h^2)^2} \rho d\rho$$

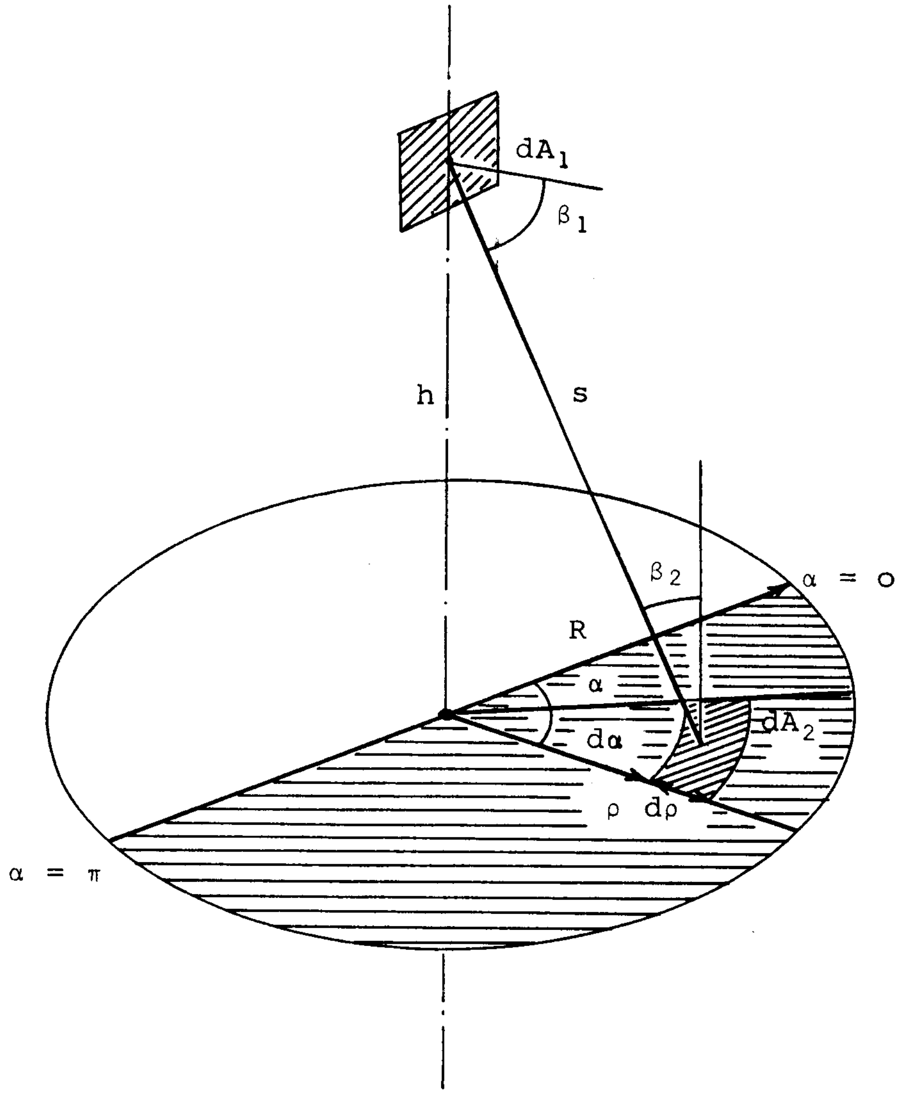
$$= 2\int_0^{R/h} \frac{1}{\left[\left(\frac{\rho}{h}\right)^2 + 1\right]^2} \left(\frac{\rho}{h}\right) d \left(\frac{\rho}{h}\right)$$

$$= -\frac{1}{2}\left[\frac{1}{\frac{\rho}{h} + 1}\right]_0^{\frac{\rho}{h} = \frac{R}{h}}$$

$$F = \frac{1}{2}\left[1 - \frac{1}{\frac{R}{h} + 1}\right] \tag{B-6}$$

SHAPE FACTOR F_2 (A_1 normal to radiator).

Let dA_1 be normal to the radiator disc and h above it. It is clear that one surface dA_1 can only "see" half of the radiator.



$$dA_2 = \rho d\alpha d\rho$$

$$s^2 = h^2 + \rho^2$$

$$\cos \beta_1 = \frac{\rho \sin \alpha}{s} \qquad \cos \beta_2 = \frac{h}{s}$$

$$F = \frac{1}{\pi}\int_0^R \int_0^\pi \frac{\rho^2}{(b^2 + \rho^2)} \, h d\rho \, \sin \alpha d\alpha$$

With the integration done for α :

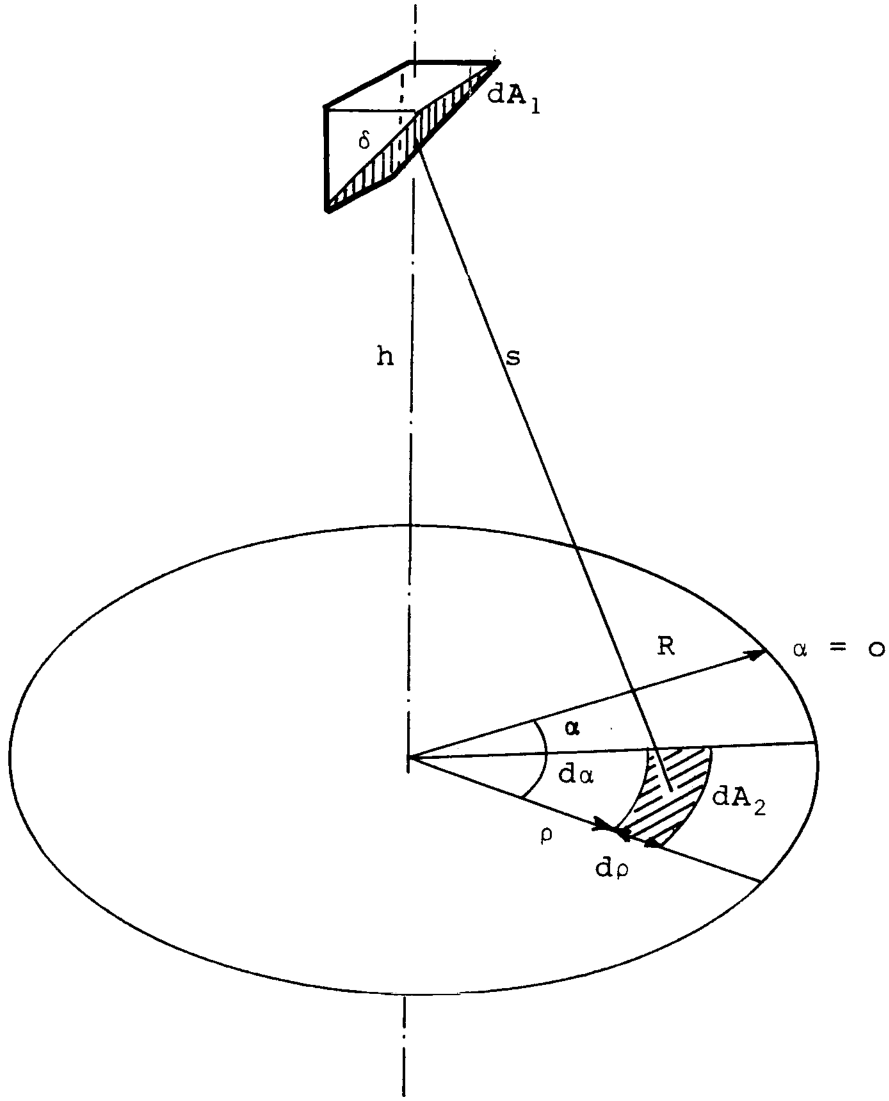
$$F_2 = \frac{2}{\pi}\int_0^R \frac{\rho^2}{(h^2 + \rho^2)^2} \, h d\rho$$

$$= \frac{2}{\pi}\int_0^{R/h} \frac{1}{\left(\frac{1}{\frac{\rho}{h}} + \frac{\rho}{h}\right)^2} \, d\left(\frac{\rho}{h}\right)$$

$$= \frac{2}{\pi}\left[-\frac{1}{2}\left\{\frac{\frac{\rho}{h}}{\left(\frac{\rho}{h}\right)^2 + 1}\right\} + \frac{1}{2}\,\text{arctg}\left(\frac{\rho}{h}\right)\right]_0^{\frac{\rho}{h} = \frac{R}{h}}$$

$$F_2 = \frac{1}{\pi}\left[\text{arctg}\left(\frac{R}{h}\right) - \left\{\frac{\frac{R}{h}}{\left(\frac{R}{h}\right)^2 + 1}\right\}\right] \qquad \text{(B-7)}$$

SHAPE FACTOR F_3 (A_1 on a cylinder surface with axis parallel radiator). Consider a cylindrical surface with its axis parallel to the radiator. A surface element dA_1 is given with an angle δ against the vertical. It can be replaced by its projections normal and parallel to the radiator.


dA$_1$
δ
h
s
R
α = o
α
dα
dA$_2$
ρ
dρ

Radiative heat received by the horizontal surface element is proportional to $F_1 \cdot (dA_1 \cdot \sin\delta)$, and it is for the vertical element porportional to $F_2 \cdot (dA_1 \cdot \cos\delta)$. For dA_1 itself we can write

$$F_{3_\delta}\, dA_1 = F_1\, (dA_1 \sin\delta) + F_2\, (dA_2 \cos\delta)$$

or

$$F_{3_\delta} = F_1 \sin\delta + F_2 \cos\delta \qquad \text{(B-8)}$$

Assuming that only the lower half of the cylindrical surface is heated up by the radiator, the following average shape factor F_3 can be obtained for the whole cylinder surface A''':

$$F_{3_\delta}\, dA_1 = (F_1 \sin\delta + F_2 \cos\delta)\, \ell\, r d\delta$$

$$= \frac{A'''}{2\pi} (F_1 \sin\delta + F_2 \cos\delta)\, d\delta$$

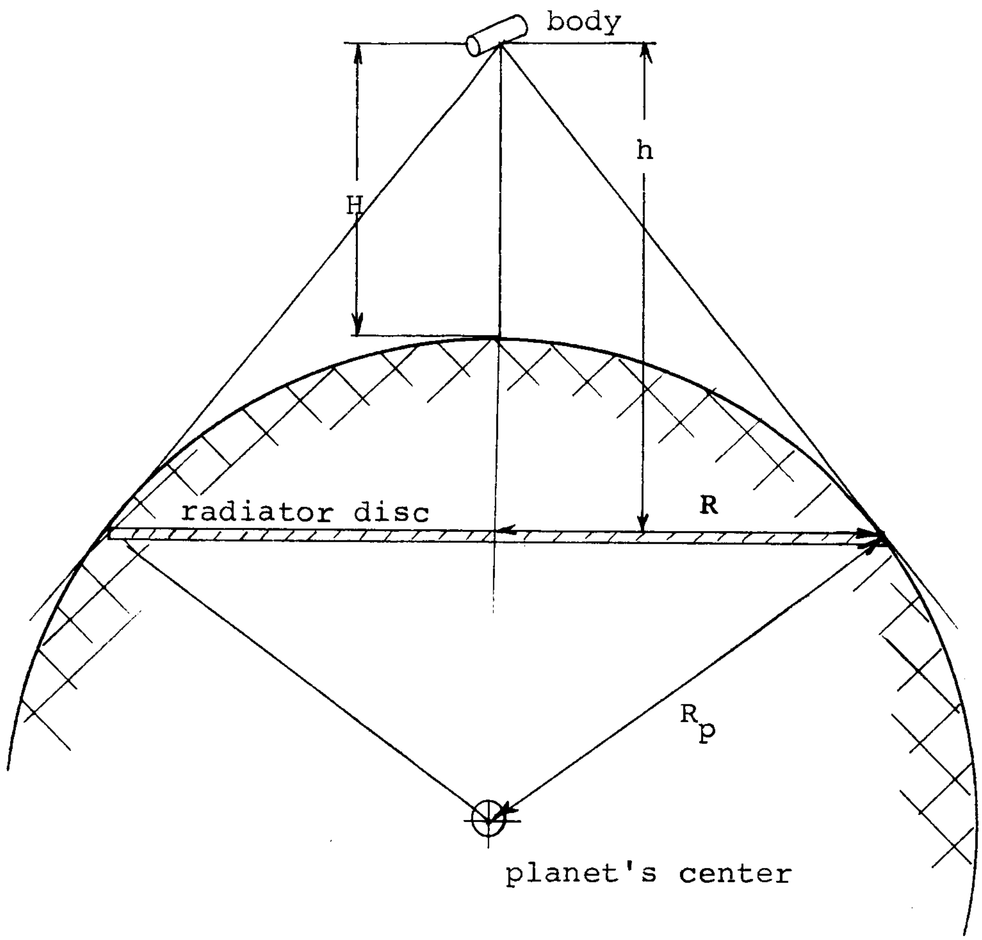
$$= F_3\, dA'''$$

$$F_3 = \frac{1}{2\pi} \left(2\, F_1 \int_0^{\pi/2} \sin\delta\, d\delta + 2\, F_2 \int_0^{\pi/2} \cos\delta\, d\delta \right)$$

$$F_3 = \frac{1}{\pi} (F_1 + F_2) \qquad \text{(B-9)}$$

NUMERICAL DETERMINATION OF THE SHAPE FACTORS

The numerical values of F_1, F_2, and F_3 for the altitudes H = 100 km above the earth's surface and for the altitude H = 110 km above Mars are determined as follows:



$$R = R_p \cdot \frac{\sqrt{(R_p + H)^2 - R_p^2}}{H + R_p}$$

$$h = \sqrt{(R_p + H)^2 - R_p^2 - R^2}$$

And so with Equation (B-10) and (B-11)

$$\frac{R}{h} = \frac{1}{\left(\frac{H}{R_p}\right) + 1} \sqrt{\frac{\left(\frac{H}{R_p}\right)^2 + 2\left(\frac{H}{R_p}\right)}{\left(\frac{H}{R_p}\right)^2 + 2\left(\frac{H}{R_p}\right) + \frac{1}{\left(\frac{H}{R_p} + 1\right)^2} - 1}}$$

This relationship is graphically represented in Figure B-1. For $H = o$, $\frac{R}{h}$ goes to infinity, and the shape factors are then

$$F_{\infty 1} = 1.0 \qquad F_{\infty 2} = 0.5 \qquad F_{\infty 3} = 0.477$$

Figure B-2 shows the form factors referenced to F_∞ for any radiator disc size, or for any true altitude above the planet's surface. The shape factors F_1 , F_2 , and F_3 used in this report are listed in Table B-1

THE COMPOSITE AREA RATIOS $\frac{A_S}{A}$ AND $\frac{A_B}{A}$:

The surface of the beam is split up into various segments according to Table B-2. The formula for the composite area ratios are finally listed in Table B-3.

PLANET	Altitude H [km]	SHAPE FACTOR		
		F_1	F_2	F_3
EARTH	100	0.966	0.383	0.429
	80	0.973	0.396	0.434
MARS	110	0.935	0.344	0.406

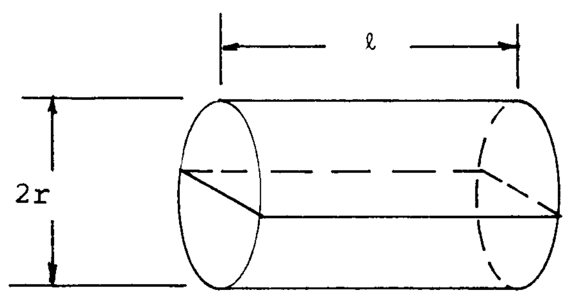
F_1 : Surface element parallel radiating surface

F_2 : Surface element normal radiating surface

F_3 : Surface element of a cylinder parallel to the radiating surface

Table B-1: Determination of the Shape Factors

Aspect Ratio $\ell/_r$	SURFACE AREA RATIO		
	$A'/_A$	$A''/_A$	$A'''/_A$
0	0.5	0	0
2	0.167	0.212	0.666
20	0.0238	0.303	0.951
200	0.00249	0.317	0.996



A	=	$2 \pi r (r + \ell)$	total surface area
A'	=	$r^2 \pi$	cross sectional area
A''	=	$2 r \ell$	projected area
A'''	=	$2 r \pi \ell = \pi A''$	cylindrical area

Table B-2: Surface Area Ratios for a Cylindrical Column of Various Aspect Ratio

Time of Day	POSITION		
	1	2	3
Midday Midnight	$\frac{A_S}{A} = \frac{A'}{A}$	$\frac{A_S}{A} = \frac{A''}{A}$	$\frac{A_S}{A} = \frac{A''}{A}$
	$\frac{A_B}{A} = F_1 \frac{A'}{A} + F_2 \frac{A'''}{A}$	$\frac{A_B}{A} = 2\ F_2 \frac{A'}{A} + F_3 \frac{A'''}{A}$	
Morning	$\frac{A_S}{A} = \frac{A''}{A}$	$\frac{A_S}{A} = \frac{A'}{A}$	$\frac{A_S}{A} = \frac{A''}{A}$
	$\frac{A_B}{A} = F_1 \frac{A'}{A} + F_2 \frac{A'''}{A}$	$\frac{A_B}{A} = 2\ F_2 \frac{A'}{A} + F_3 \frac{A'''}{A}$	

Table B-3: Determination of the Composite Surface Area Ratios $\frac{A_S}{A}$ and $\frac{A_B}{A}$

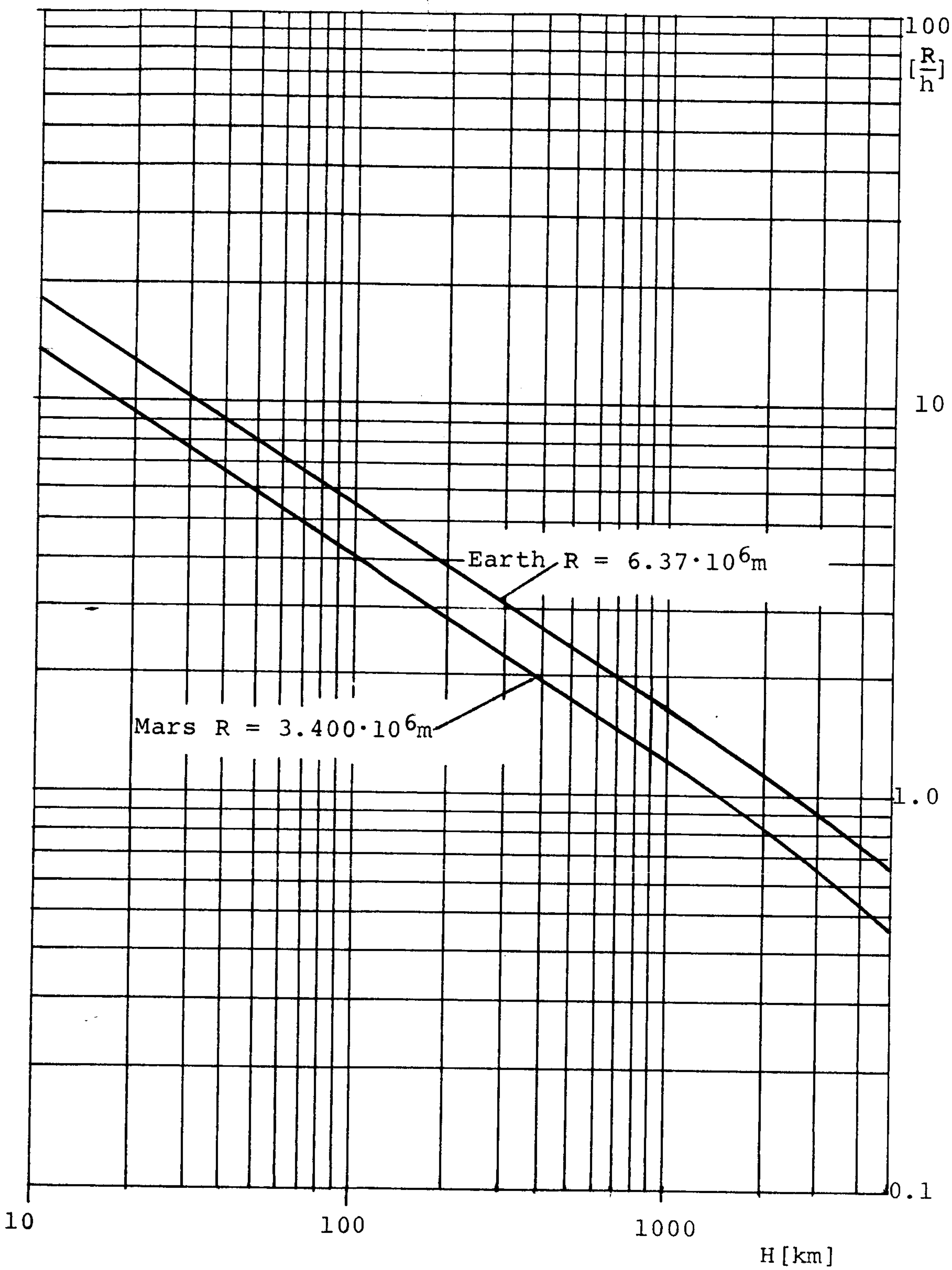


Figure B-1. Radiator Disc Size Versus Altitude Above Planet

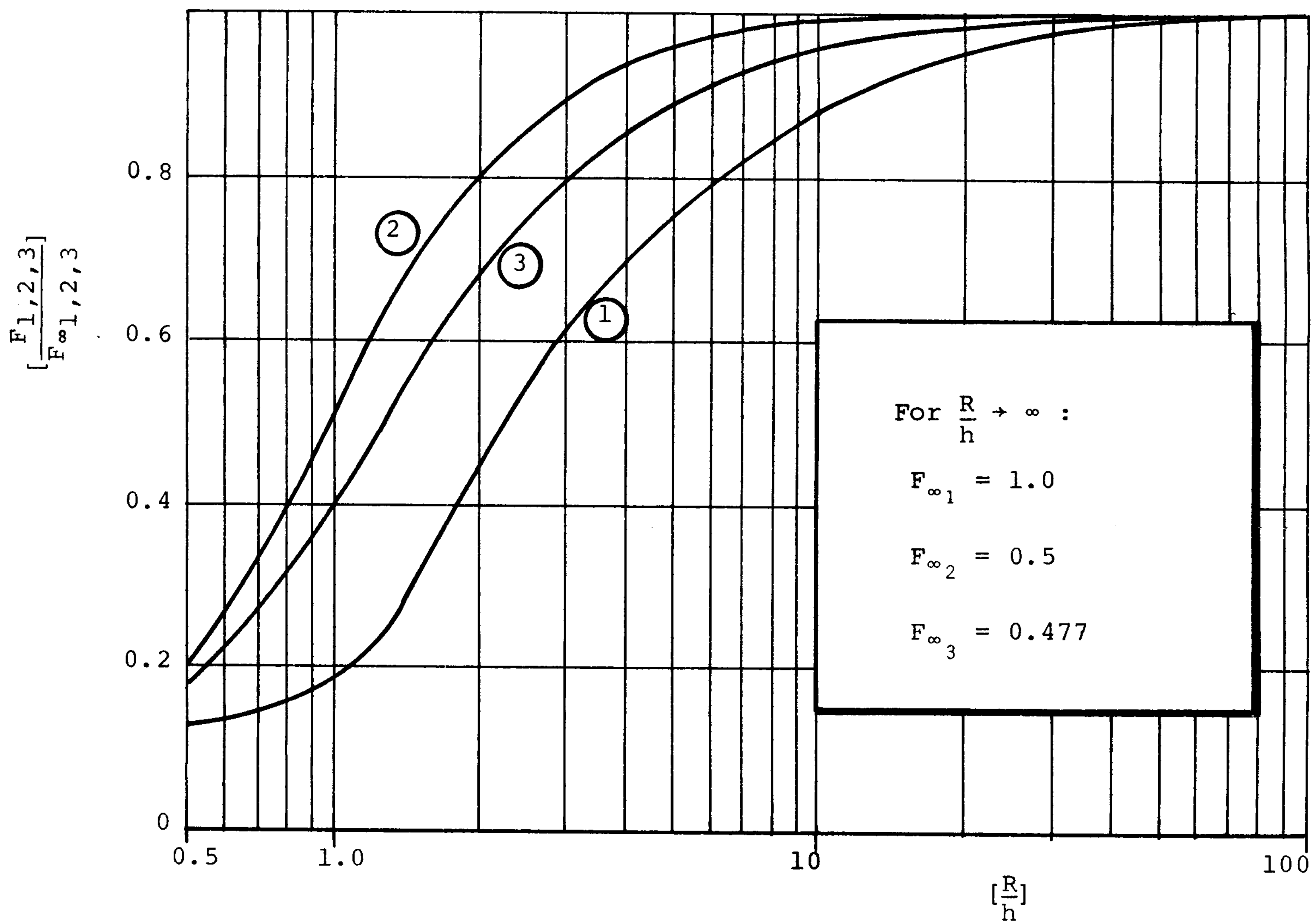


Figure B-2. Radiative Shape Factor Versus Radiator Disc Size

APPENDIX C

A SUMMARY OF EMISSIVITY AND SOLAR ABSORPTIVITY COEFFICIENTS

NOTE TO TABLE C-1:

Assuming no convective or conductive losses, the solar absorptance of merit is:

$$\alpha^1 = \alpha - \varepsilon_C/_\beta$$

with:

α total solar absorptance averaged over the direction of the irradiation

ε_C total hemispherical emittance

β ratio of irradiation G to the black body emissive power $\sigma T_C{}^4$ of the collector at temperature T_C

TABLE 5.2

EVALUATION OF MATERIALS FOR SOLAR COLLECTORS IRRADIATED BY 442 Btu/hr ft^2 FROM A 10,000 R PLANKIAN RADIATOR

No.	Surface	Index No. in Sect. 2	α	T_c = 555R			T_c = 1,000R		
				ϵ_c	α/ϵ_c	α'	ϵ_c	α/ϵ_c	α'
1	Silicon coated aluminum 10μ polycrystalline film on aluminum foil.		0.522*	0.12*	4.35	0.477	0.12*	4.35	0.057
2	Silicon solar cell, International Rectifier Corp. Approx. 1 mm thick on electroless nickel plate substrate. Boron-doped surface.	94	0.938	0.316	2.97	0.822	0.497	1.89	-0.992
3	Chromium plate 0.1 mil thick on 0.5 mil nickel plate on 321 stainless steel exposed to JP4 combustion products 50 hours at 1,100 F.	15	0.778	0.150	5.18	0.723	0.182	4.27	0.071
4	Tabor Solar Collector Chemical Treatment 110-30 on nickel-plated copper.	17	0.853	0.049	17.4	0.835	0.078	10.9	0.550
5*	Tabor Solar Collector Chemical Treatment 125-30 on nickel-plated copper.	18	0.851	0.108	7.87	0.811	0.270	3.15	-0.195
6	410 Stainless Steel heated to 1,300 F in air.	48	0.764	0.130	5.88	0.716	0.180	4.24	0.064
7	Titanium 75A (AMS 4901) heated 300 hours at 850 F in air.	60	0.798	0.211	3.78	0.720	0.294	2.72	-0.342
8	Titanium C-110 M (AMS 4908) heated 100 hours at 800 F in air.	62	0.524	0.162	3.24	0.464	0.202	2.59	-0.261
9	Titanium C-110 M (AMS 4908) heated 300 hours at 850 F in air.	63	0.768	0.198	3.88	0.695	0.246	3.12	-0.186
10	Titanium vapor coated on bright side of Reynold's Wrap Aluminum foil, 80 to 100 microns thick heated 3 hours at 750 F in air.	67	0.746	0.138	5.40	0.695	0.212	3.52	-0.077
11	Tabor Solar Collector Chemical Treatment of galvanized iron.	71	0.885	0.122	7.25	0.840	0.264	3.36	-0.140
12	Ebanol C on copper treated 5 min. at 198 F in 219 F boiling point solution.		0.908	0.11	8.25	0.867			
13	Ebanol S on steel treated 15 min. in a 286 F boiling solution.		0.848	0.10	8.40	0.812			
14	Ideal Black Body		1.000	1.00	1.00	.632	1.00	1.00	-2.89

*Estimated from optical properties in Table 5.1 and Figure 2.3.2 with use of Equation (5.15).

Source: BASIC STUDIES ON THE USE AND CONTROL OF SOLAR ENERGY
Edwards/Nelson/Roddick/Gier
UCLA Report 60-93 Part V

Table C-1: Summary of Emissivity and Absorptivity Coefficients

TABLE 4-1. EMISSIVITY AND SOLAR ABSORPTIVITY OF METALS AND PIGMENTS

Metal or pigment	Emissivity ε						Solar absorptivity
	−300°F	0°F	300°F	900°F	1500°F	1800°F	
Mild steel:							
As received	0.12	0.12	0.13	0.30	0.34	....	0.14
Polished	0.07	0.08	0.14	0.32	0.35	0.31	0.41
Stainless steel type 301:							
As received	0.21	0.21	0.22	0.27	0.39	....	0.44
Polished	0.15	0.16	0.17	0.23	0.48	0.77	0.38
Inconel X:							
As received	0.21	0.21	0.21	0.22	0.30	....	0.44
Polished	0.19	0.19	0.20	0.20	0.53	0.62	0.39
7075 Aluminum:							
As received	0.08	0.10	0.13	0.17	....	....	0.59
Polished	0.06	0.08	0.09	0.12	....	....	0.34
2024 Aluminum:							
As received	0.08	0.09	0.11	0.14	....	....	0.49
Polished	0.08	0.09	0.10	0.14	....	....	0.25
Magnesium:							
As received	0.13	0.15	0.17	....	....	....	0.59
Polished	0.11	0.12	0.14	....	....	....	0.31
Molybdenum	....	....	....	0.75	0.18	0.30	0.43
Lamp black	....	0.96	....	0.97	....	....	0.97
Copper oxide (Cu_2O_2)	....	0.95	....	0.67	....	....	0.73
Red iron oxide (Fe_2O_3)	....	0.96	....	0.70	....	....	0.74
White lead ($PbCO_3$)	....	0.89	....	0.71	....	....	0.12

Source: STRUCTURAL DESIGN OF MISSILES AND SPACECRAFT
L. H. Abraham
McGraw Hill

Table C-2: Summary of Emissivity and Absorptivity Coefficients

APPENDIX D

DETERMINATION OF THE HEAT FILM COEFFICIENT h FOR A HEAT TRANSFER DUE TO FORCED CONVECTION

As space environment, points above the planet earth and Mars are chosen with the altitudes $H = 80$ km and 100 km for the earth and $H = 110$ km for Mars. A descent velocity $V = 250 \frac{m}{s}$ at an altitude of 100 km above the earth's surface represents approximately the upper possible limit for subsonic operation of a high altitude decelerator as described in Reference 8. At 80 km above the earth the atmospheric temperature reaches its minimum and the decelerator has approximately $V = 80 \frac{m}{s}$ descent velocity. For Mars, the atmospheric model No. 2 for a mean atmosphere is chosen as described in Reference 4 (surface pressure 25 mb, surface temperature 250°K). At an altitude 110 km above the Martian surface the atmospheric density is the same as 80 km above the earth.

The heat transfer coefficient can generally be expressed as a function of Nusselt number

$$h = \frac{k\ Nu_d}{d} \quad \text{or} \quad h = \frac{k\ Nu_\ell}{\ell} \qquad \text{(D-1)}$$

with:

d: column diameter

ℓ: column length

While the thermal conductivity k of the gas is a function of temperature only, the Nusselt number generally is a function of the particular flow condition and it is expressed in terms of Prandtl number and Reynolds number:

$$Pr = \frac{C_p \cdot \eta}{k} \approx 0.7 \qquad \text{(D-2)}$$

$$Re_d = \frac{d\ V\ \rho}{\eta} \qquad Re_\ell = \frac{\ell\ V\ \rho}{\eta} \qquad \text{(D-3)}$$

It is assumed that the column moves with a velocity V vertically towards the planet. According to Reference 5, page 176, a cylindrical tube in laminar parallel flow (column in position 1) has a Nusselt number (based on column length).

$$Nu_{\ell} = 0.332 \sqrt[3]{P_r} \sqrt{Re_{\ell}} \tag{D-4}$$

$$Nu_{\ell} = 0.295 \sqrt{Re_{\ell}}$$

The positions 2 and 3 of the column represent the case of a cylinder in a cross flow condition. Reference 5, page 242, given then for the Nusselt number (based on column diameter).

$$Nu_d = 0.43 + 0.48 \sqrt{Re_d} \tag{D-5}$$

for

$$Re_d < 4000$$

This short analysis neglects aspect ratio effects of the cylinder.

This simplified analysis assumes

$$d = \ell = 0.1 \text{ m}$$

The importance of the forced convection term being moderate, column lengths or column diameters larger than 0.1 m yield smaller film coefficients h, decreasing the influence of the forced convection term on the equilibrium temperature. Rarefied gas effect on the flow situation are neglected.

APPENDIX E

NUMERICAL EXAMPLE

Reference 8 contains a design study for a high altitude parachute of 1 lb = 4.44 N total weight. Figure E-1 shows the concept of the X-brace parachute. The square canopy is deployed and stiffened by an X-shaped inflated bracing system with the following design characteristics:

Material:	aluminized Mylar
Thickness:	$\Delta = 2 \cdot 0.25$ mil equivalent to 0.5 mil
Density:	$\rho_s = 1.38 \cdot 10^3 \frac{kg}{m^3}$
Solar Absorptivity:	$\alpha = 0.2$ (at a wavelength $\lambda = 0.48\mu$)
Emissivity:	$\varepsilon = 0.07$ (at a wavelength $\lambda \sim 10\mu$)
	$\frac{\alpha}{\varepsilon} = 2.86$
Brace Length:	$L = 6.06$ m

The brace legs are designed to resist a compressive load due to the aerodynamic loading of the canopy. This load is related to the weight of the whole system as follows:

$$P = \frac{5\sqrt{2}}{48} W_{tot} = 0.655 \text{ N}$$

to be carried over the half length $\ell = 3.03$ m of the brace. For minimum weight we assume

$$k = 1$$

and with Equation (19):

$$\sigma = \sqrt[3]{\frac{p^2}{\ell^2} \cdot \frac{E}{\Delta^2}} = 1.033 \cdot 10^6 \frac{N}{m^2}$$

with Equation (20):

$$AR = \pi \sqrt{\frac{E}{\sigma}} = 190.4$$

or

$$r = \frac{\ell}{AR} = 1.59 \cdot 10^{-2} \text{ m}$$

According to Equation (18) the following internal pressure is required:

$$p = \frac{\sigma \Delta}{r} = 0.825 \cdot 10^3 \frac{N}{m^2} = 6.19 \text{ mmHg} = 0.1195 \text{ [psi]}$$

The following operating environment is chosen:

planet	:	earth
altitude	:	100 km
descent velocity	:	250 $\frac{m}{s}$
daytime	:	midday, position 2

Figure 3 gives the following equilibrium temperature:

$$T_o = 145°C = 418°K$$

According to Reference 8 a minimum packing volume has to be aimed at due to the limited space available in the carrier vehicle. If liquid nitrogen is ruled out for the present application, there is only Freon 11 left as an alternative. Figure 10 gives then for

$T = 145°C$ and $p_d = 6.19$ mmHg (0.1195 psi):

$$v = \frac{V_p}{V_d} = 0.0022\%$$

This operating point is well away from the saturation limit (at $T = 66°C$ for the same pressure). The total amount of Freon needed for both cylindrical braces is then:

$$m = 2\ V_d \cdot \frac{p_d}{RT_d} = 0.000315 \text{ kg}$$

with a liquid volume to be packed at

$$V_p = 2.11 \cdot 10^{-7} \text{ m}^3 = 0.211 \text{ cm}^3$$

which is an almost negligible amount of the total packing volume in the parachute container. (These values can be reduced by coning the braces.) The total weight of the brace system is with Equation (22) or (23):

$$W_{brace} = 4\ (P \cdot \ell)\ g\ (2\ \frac{\rho_s}{\sigma} + \frac{1}{RT})$$

$$= 0.179 \text{ N}$$

or 4% of the total weight of the whole system.

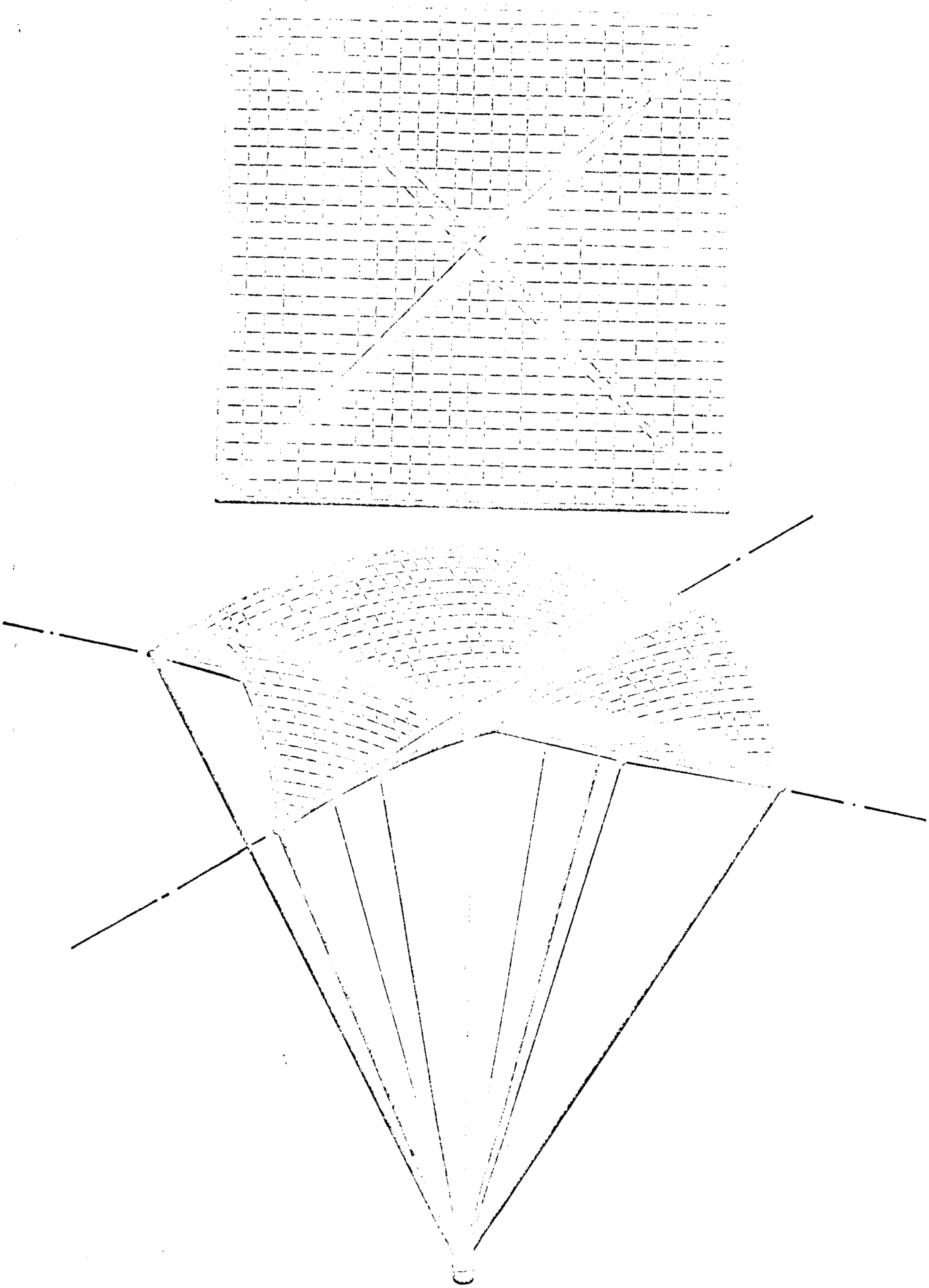


Figure E-1: The X-Brace Parachute